DIAGNOSTIC TEST ANSWER SHEET

1. __________	21. __________	41. __________
2. __________	22. __________	42. __________
3. __________	23. __________	43. __________
4. __________	24. __________	44. __________
5. __________	25. __________	45. __________
6. __________	26. __________	46. __________
7. __________	27. __________	47. __________
8. __________	28. __________	48. __________
9. __________	29. __________	49. __________
10. __________	30. __________	50. __________
11. __________	31. __________	51. __________
12. __________	32. __________	52. __________
13. __________	33. __________	53. __________
14. __________	34. __________	54. __________
15. __________	35. __________	55. __________
16. __________	36. __________	56. __________
17. __________	37. __________	57. __________
18. __________	38. __________	58. __________
19. __________	39. __________	59. __________
20. __________	40. __________	60. __________

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3.1 Getting Started!

Taking the Diagnostic Test helps you assess your strengths and weaknesses as you begin preparing for the AP Calculus BC exam. The questions in the Diagnostic Test contain both multiple-choice and open-ended questions. They are arranged by topic, and designed to review concepts tested on the AP Calculus BC exam. All questions in the diagnostic test should be done without the use of a graphing calculator, except in a few cases where you need to find the numerical value of a logarithmic or exponential function.

3.2 Diagnostic Test

Chapter 5

1. A function f is continuous on $[-2, 0]$ and some of the values of f are shown below.

x	-2	-1	0
f	4	b	4

If $f(x) = 2$ has no solution on $[-2, 0]$, then b could be

(A) 3

(B) 2

(C) 1

(D) 0

(E) -2

2. Evaluate $\lim_{x\to-\infty} \frac{\sqrt{x^2-4}}{2x}$.

3. If

$$h(x) = \begin{cases} \sqrt{x} & \text{if } x > 4 \\ x^2 - 12 & \text{if } x \le 4 \end{cases} \quad \text{find } \lim_{x\to4} h(x).$$

4. If $f(x) = |2xe^x|$, what is the value of $\lim_{x\to0^+} f'(x)$?

Chapter 6

5. If $f(x) = -2\csc(5x)$, find $f'\left(\frac{\pi}{6}\right)$.

6. Given the equation $y = (x+1)(x-3)^2$, what is the instantaneous rate of change of y at $x = -1$?

7. What is $\lim_{\Delta x\to0} \frac{\tan\left(\frac{\pi}{4}+\Delta x\right) - \tan\left(\frac{\pi}{4}\right)}{\Delta x}$?

8. Evaluate $\lim_{x\to\pi} \frac{e^x - e^\pi}{x^e - \pi^e}$.

Chapter 7

9. The graph of f is shown in Figure D-1. Draw a possible graph of f' on (a, b).

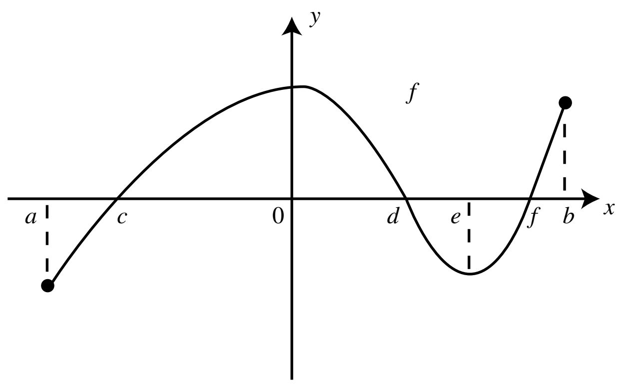

Figure D-1

10. The graph of the function g is shown in Figure D-2. Which of the following is true for g on (a, b)?

I. g is monotonic on (a, b).

II. g' is continuous on (a, b).

III. $g'' > 0$ on (a, b).

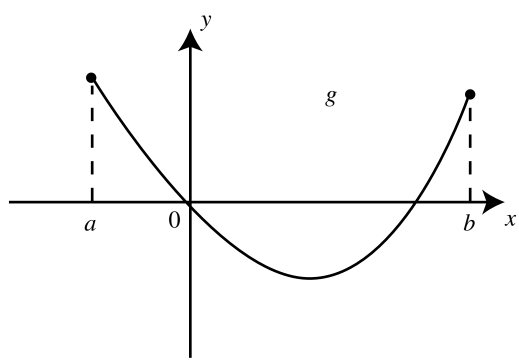

Figure D-2

11. The graph of f is shown in Figure D-3 and f is twice differentiable, which of the following statements is true?

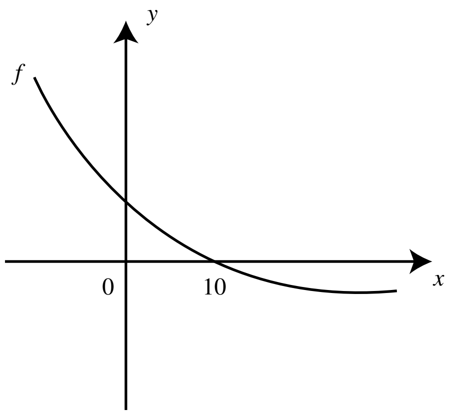

Figure D-3

(A) $f(10) < f'(10) < f''(10)$

(B) $f''(10) < f'(10) < f(10)$

(C) $f'(10) < f(10) < f''(10)$

(D) $f'(10) < f''(10) < f(10)$

(E) $f''(10) < f(10) < f'(10)$

12. The graph of f', the derivative of f, is shown in Figure D-4. At what value(s) of x is the graph of f concave up?

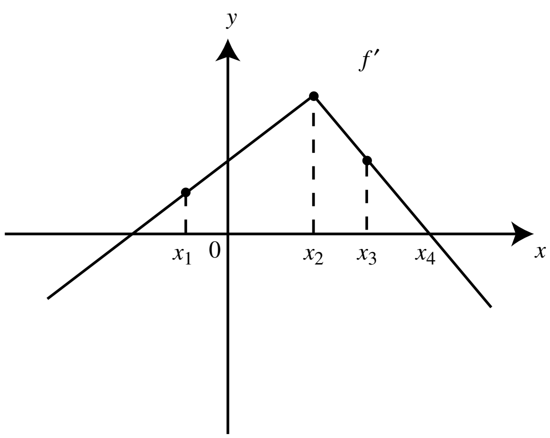

Figure D-4

13. How many points of inflection does the graph of $y = \sin(x^2)$ have on the interval $[-\pi, \pi]$?

14. If $g(x) = \int_a^x f(t)\,dt$ and the graph of f is shown in Figure D-5, which of the graphs in Figure D-6 on the next page is a possible graph of g?

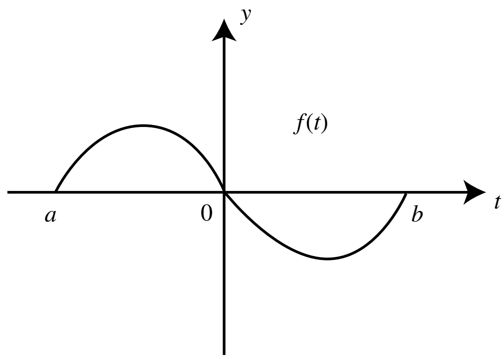

Figure D-5

15. The graphs of f', g', p', and q' are shown in Figure D-7 on the next page. Which of the functions f, g, p, or q have a point of inflection on (a, b)?

16. Find the rectangular equation of the curve defined by $x = 1 + e^{-t}$ and $y = 1 + e^{t}$.

Chapter 8

17. When the area of a square is increasing four times as fast as the diagonals, what is the length of a side of the square?

18. If $g(x) = |x^2 - 4x - 12|$, which of the following statements about g is/are true?

I. g has a relative maximum at $x = 2$.

II. g is differentiable at $x = 6$.

III. g has a point of inflection at $x = -2$.

Chapter 9

19. Given the equation $y = \sqrt{x-1}$, what is an equation of the normal line to the graph at $x = 5$?

20. What is the slope of the tangent to the curve $y = \cos(xy)$ at $x = 0$?

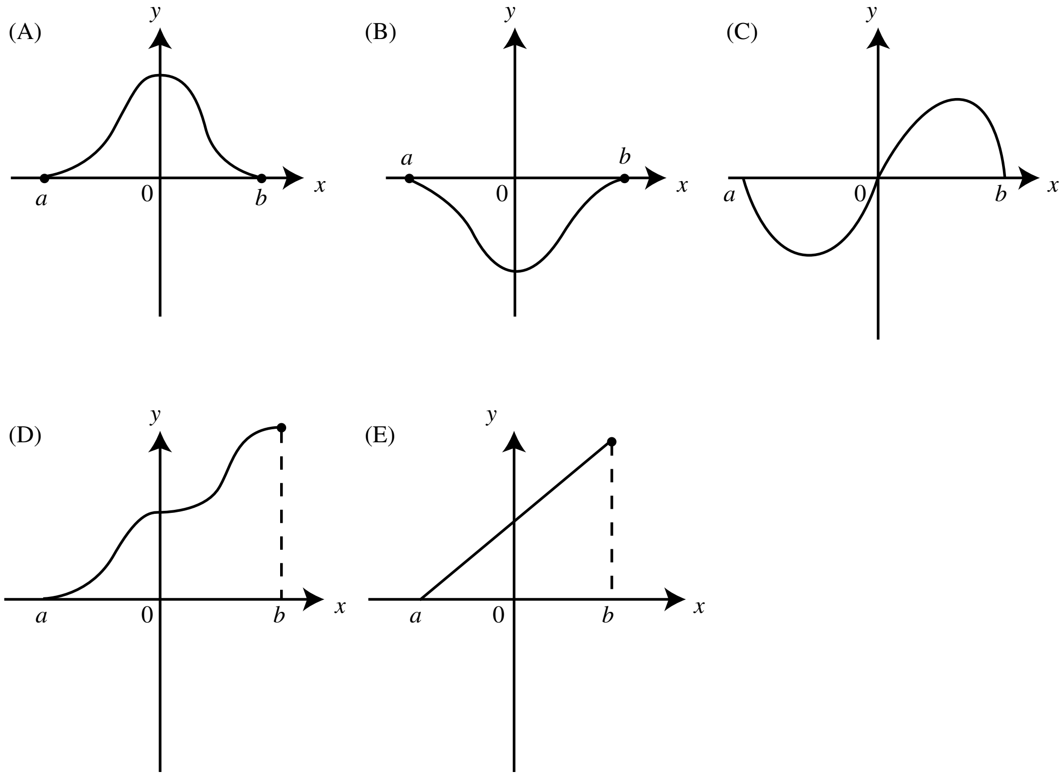

Figure D-6

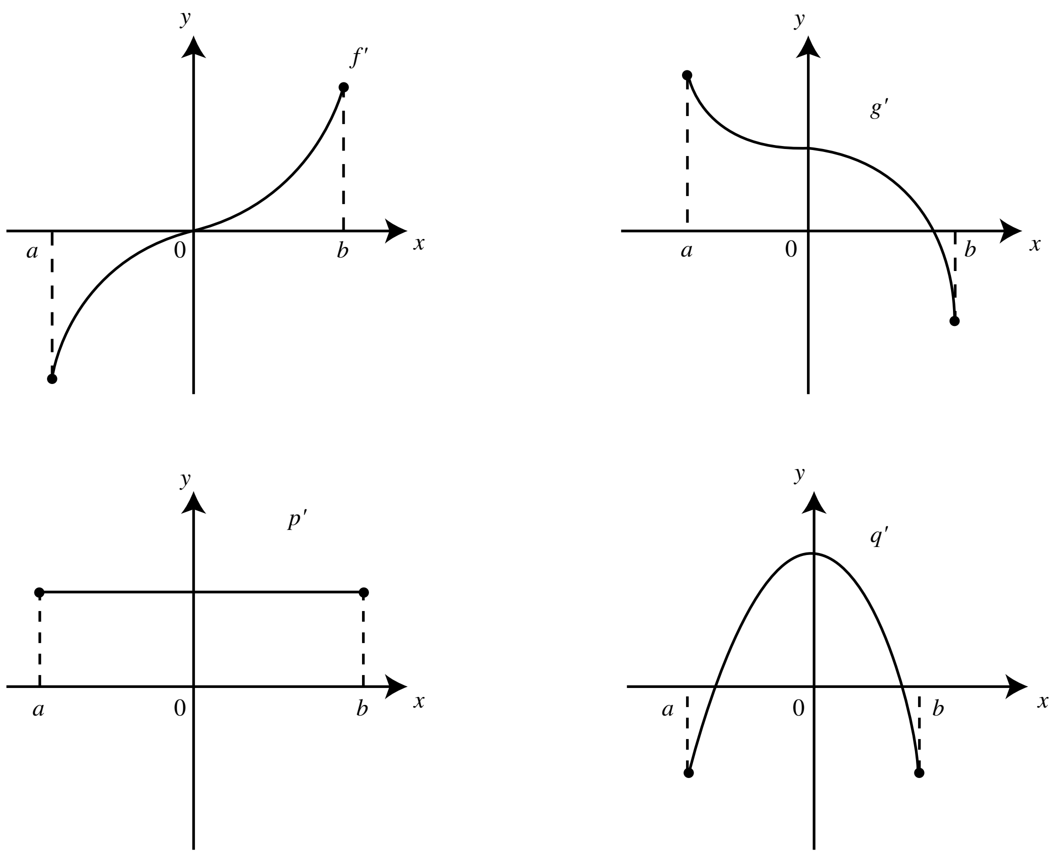

Figure D-7

21. The velocity function of a moving particle on the x-axis is given as $v(t) = t^2 - t$. For what values of t is the particle's speed decreasing?

22. The velocity function of a moving particle is $v(t) = \frac{t^3}{3} - 2t^2 + 5$ for $0 \le t \le 6$. What is the maximum acceleration of the particle on the interval $0 \le t \le 6$?

23. Write an equation of the normal line to the graph of $f(x) = x^3$ for $x \ge 0$ at the point where $f'(x) = 12$.

24. At what value(s) of x do the graphs of $f(x) = \frac{\ln x}{x}$ and $y = -x^2$ have perpendicular tangent lines?

25. Given a differentiable function f with $f\left(\frac{\pi}{2}\right) = 3$ and $f'\left(\frac{\pi}{2}\right) = -1$. Using a tangent line to the graph at $x = \frac{\pi}{2}$, find an approximate value of $f\left(\frac{\pi}{2} + \frac{\pi}{180}\right)$.

26. An object moves in the plane on a path given by $x = 4t^2$ and $y = \sqrt{t}$. Find the acceleration vector when $t = 4$.

27. Find the equation of the tangent line to the curve defined by $x = 2t + 3$, $y = t^2 + 2t$ at $t = 1$.

Chapter 10

28. Evaluate $\int \frac{1 - x^2}{x^2}\, dx$.

29. If $f(x)$ is an antiderivative of $\frac{e^x}{e^x + 1}$ and $f(0) = \ln(2)$, find $f(\ln 2)$.

30. Find the volume of the solid generated by revolving about the x-axis on the region bounded by the graph of $y = \sin 2x$ for $0 \le x \le \pi$ and the line $y = \frac{1}{2}$.

31. Evaluate $\int_2^5 \frac{1}{x^2 + 2x - 3}\, dx$.

32. Evaluate $\int x^2 \cos x\, dx$.

Chapter 11

33. Evaluate $\int_1^4 \frac{1}{\sqrt{x}}\, dx$.

34. If $\int_{-1}^{k} (2x - 3)\, dx = 6$, find k.

35. If $h(x) = \int_{\pi/2}^{x} \sqrt{\sin t}\, dt$, find $h'(\pi)$.

36. If $f'(x) = g(x)$ and g is a continuous function for all real values of x, then $\int_0^2 g(3x)\, dx$ is

(A) $\frac{1}{3} f(6) - \frac{1}{3} f(0)$.
(B) $f(2) - f(0)$.
(C) $f(6) - f(0)$.
(D) $\frac{1}{3} f(0) - \frac{1}{3} f(6)$.
(E) $3 f(6) - 3 f(0)$.

37. Evaluate $\int_{\pi}^{x} \sin(2t)\, dt$.

38. If a function f is continuous for all values of x, which of the following statements is/are always true?

I. $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$

II. $\int_a^b f(x)dx = \int_a^c f(x)dx - \int_c^b f(x)dx$

III. $\int_b^c f(x)dx = \int_b^a f(x)dx - \int_c^a f(x)dx$

39. If $g(x) = \int_{\pi/2}^{x} 2 \sin t\, dt$ on $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$, find the value(s) of x where g has a local minimum.

40. Evaluate $\int_0^{\infty} e^{-x}\,dx$.

Chapter 12

41. The graph of the velocity function of a moving particle is shown in Figure D-8. What is the total distance traveled by the particle during $0 \le t \le 6$?

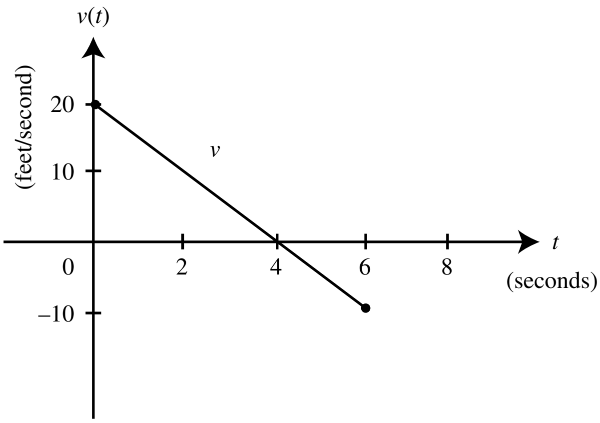

Figure D-8

42. The graph of f consists of four line segments, for $-1 \le x \le 5$ as shown in Figure D-9. What is the value of $\int_{-1}^{5} f(x)\,dx$?

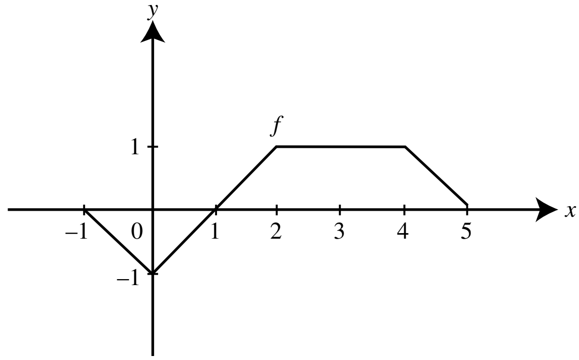

Figure D-9

43. Find the area of the region enclosed by the graph of $y = x^2 - x$ and the x-axis.

44. If $\int_{-k}^{k} f(x)\,dx = 0$ for all real values of k, then which of the graphs in Figure D-10 could be the graph of f?

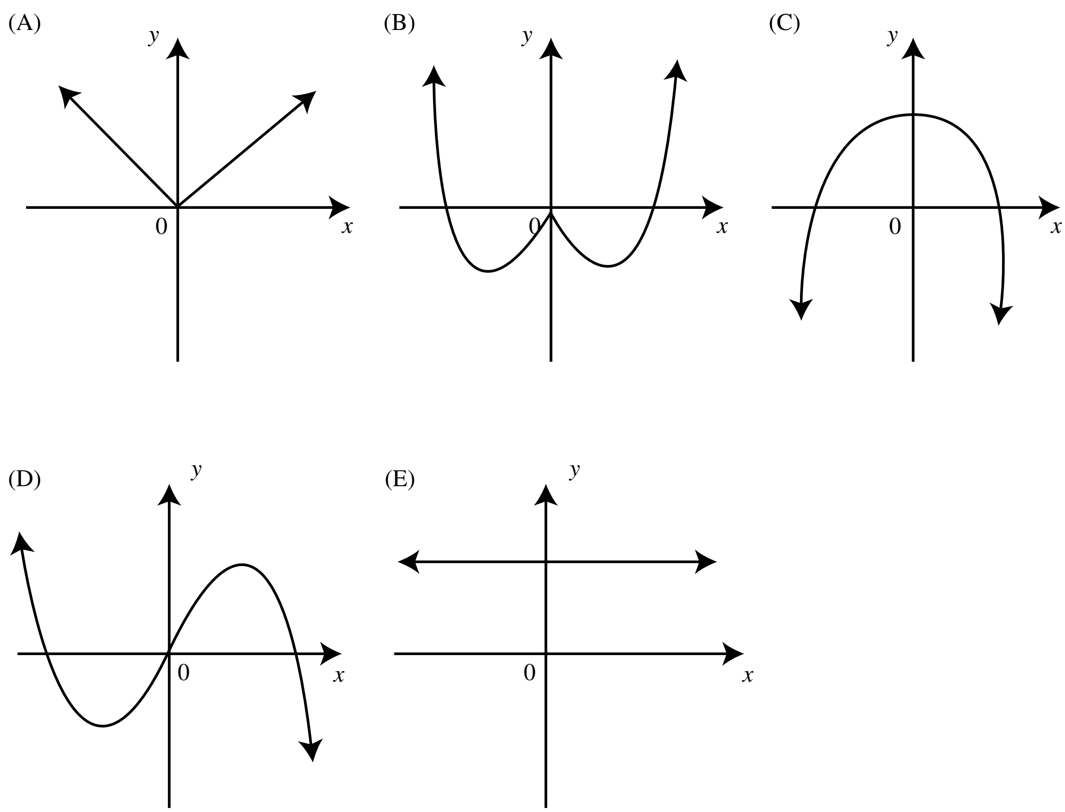

Figure D-10

45. The area under the curve $y=\sqrt{x}$ from $x=1$ to $x=k$ is 8. Find the value of k.

46. For $0 \leq x \leq 3\pi$, find the area of the region bounded by the graphs of $y=\sin x$ and $y=\cos x$.

47. Let f be a continuous function on [0, 6] that has selected values as shown below:

x	0	1	2	3	4	5	6
$f(x)$	1	2	5	10	17	26	37

Using three midpoint rectangles of equal widths, find an approximate value of

$$\int_0^6 f(x)dx.$$

48. Find the total area bounded by the curves $r=2\cos\theta$ and $r=2\sin\theta$.

49. Determine the length of the curve defined by $x=3t-t^3$ and $y=3t^2$ from $t=0$ to $t=2$.

Chapter 13

50. What is the average value of the function $y=e^{-4x}$ on $[-\ln 2, \ln 2]$?

51. If $\frac{dy}{dx}=2\sin x$ and at $x=\pi$, $y=2$, find a solution to the differential equation.

52. Water is leaking from a tank at the rate of $f(t)=10\ln(t+1)$ gallons per hour for $0\leq t\leq 10$, where t is measured in hours. How many gallons of water have leaked from the tank after exactly 5 hours?

53. Carbon-14 has a half-life of 5730 years. If y is the amount of Carbon-14 present and y decays according to the equation $\frac{dy}{dt}=ky$, where k is a constant and t is measured in years, find the value of k.

54. What is the volume of the solid whose base is the region enclosed by the graphs of $y=x^2$ and $y=x+2$ and whose cross sections perpendicular to the x-axis are squares?

55. The growth of a colony of bacteria in a controlled environment is modeled by $\frac{dP}{dt}=.35P\left(1-\frac{P}{4000}\right)$. If the initial population is 100, find the population when $t=5$.

56. If $\frac{dy}{dx}=\frac{-y}{x^2}$ and $y=3$ when $x=2$, approximate y when $x=3$ using Euler's Method with a step size of 0.5.

Chapter 14

57. Determine whether the series $\sum_{n=0}^{\infty}\frac{3}{(n+1)^4}$ converges or diverges.

58. For what values of x does the series $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$ converge absolutely?

59. Find the Taylor series expansion of $f(x)=\frac{1}{x}$ about the point $x=2$.

60. Find the MacLaurin series for e^{-x^2}.

3.3 Answers to Diagnostic Test

1. A

2. $-\frac{1}{2}$

3. Does not exist

4. 2

5. $-20\sqrt{3}$

6. 16

7. 2

8. $\frac{e^{\pi-1}}{\pi^{e-1}}$

9. See Figure DS-2 in solution

10. II & III

11. C

12. $x < x_2$

13. 8

14. A

15. q

16. $y = \frac{x}{x-1}$

17. $2\sqrt{2}$

18. I

19. $y = -4x + 22$

20. 0

21. $\left(\frac{1}{2}, 1\right)$

22. 12

23. $y = \frac{-1}{12}x + \frac{49}{6}$

24. 1.370

25. 2.983

26. $\left\langle 8, -\frac{1}{32} \right\rangle$

27. $y = 2x - 7$

28. $\frac{-1}{x} - x + C$

29. $\ln 3$

30. 1.503

31. $\frac{1}{4}\ln\left(\frac{5}{2}\right)$

32. $x^2 \sin x + 2x \cos x - 2 \sin x + C$

33. 2

34. $\{-2, 5\}$

35. 0

36. A

37. $\frac{-1}{2}\cos(2x) + \frac{1}{2}$

38. I & III

39. 2π

40. 1

41. 50 feet

42. 2

43. $\frac{1}{6}$

44. D

45. $13^{2/3}$

46. 5.657

47. 76

48. $\frac{\pi}{2} - 1$

49. 14

50. $\frac{255}{128 \ln 2}$

51. $y = -2\cos x$

52. 57.506

53. $\frac{-\ln 2}{5730}$

54. $\frac{81}{10}$

55. 514.325

56. 2.415

57. Converges

58. $-1 < x < 1$

59. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x-2)^n$

60. $\sum_{n=0}^{\infty} \frac{(-1)^n \left(x^{2n}\right)}{n!}$

3.4 Solutions to Diagnostic Test

Chapter 5

1. See Figure DS-1.
If $b=2$, then $x=-1$ would be a solution for $f(x)=2$.
If $b=1$, 0 or–2, $f(x)=2$ would have two solutions.
Thus, $b=3$, choice (A).

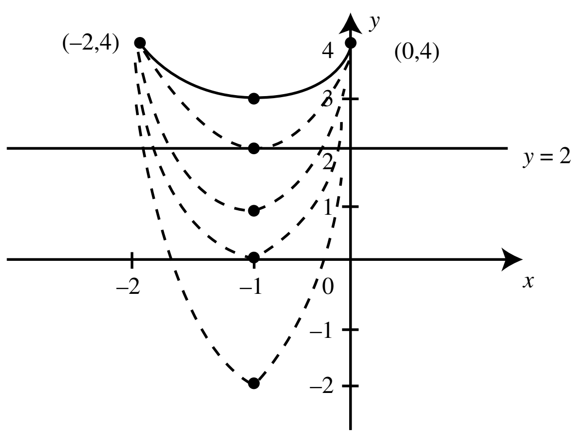

Figure DS-1

2. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2-4}}{2x}=\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2-4}/-\sqrt{x^2}}{2x/-\sqrt{x^2}}$

(Note: as $x\to-\infty$, $x=-\sqrt{x^2}$.)

$=\lim\limits_{x\to-\infty}\dfrac{-\sqrt{(x^2-4)/x^2}}{2}$

$=\lim\limits_{x\to-\infty}\dfrac{-\sqrt{1-(4/x^2)}}{2}$

$=-\dfrac{\sqrt{1}}{2}=-\dfrac{1}{2}$

3. $h(x)=\begin{cases}\sqrt{x} & \text{if } x>4\\ x^2-12 & \text{if } x\le 4\end{cases}$

$\lim\limits_{x\to4^+}h(x)=\lim\limits_{x\to4^+}\sqrt{x}=\sqrt{4}=2$

$\lim\limits_{x\to4^-}h(x)=\lim\limits_{x\to4^-}(x^2-12)=(4^2-12)=4$

Since $\lim\limits_{x\to4^+}h(x)\neq\lim\limits_{x\to4^-}h(x)$, thus $\lim\limits_{x\to4}h(x)$ does not exist.

4. $f(x)=|2xe^x|=\begin{cases}2xe^x & \text{if } x\ge 0\\ -2xe^x & \text{if } x<0\end{cases}$

If $x\ge 0$, $f'(x)=2e^x+e^x(2x)=2e^x+2xe^x$

$\lim\limits_{x\to0^+}f'(x)=\lim\limits_{x\to0^+}(2e^x+2xe^x)=2e^0+0=2$

Chapter 6

5. $f(x)=-2\csc(5x)$

$f'(x)=-2(-\csc 5x)[\cot(5x)](5)$
$=10\csc(5x)\cot(5x)$

$f'\left(\dfrac{\pi}{6}\right)=10\csc\left(\dfrac{5\pi}{6}\right)\cot\left(\dfrac{5\pi}{6}\right)$
$=10(2)(-\sqrt{3})=-20\sqrt{3}$

6. $y=(x+1)(x-3)^2$;

$\dfrac{dy}{dx}=(1)(x-3)^2+2(x-3)(x+1)$
$=(x-3)^2+2(x-3)(x+1)$

$\left.\dfrac{dy}{dx}\right|_{x=-1}=(-1-3)^2+2(-1-3)(-1+1)$
$=(-4)^2+0=16$

7. $f'(x_1)=\lim\limits_{\Delta x\to0}\dfrac{f(x_1+\Delta x)-f(x_1)}{\Delta x}$

Thus, $\lim\limits_{\Delta x\to0}\dfrac{\tan\left(\frac{\pi}{4}+\Delta x\right)-\tan\left(\frac{\pi}{4}\right)}{\Delta x}$

$=\dfrac{d}{dx}(\tan x)$ at $x=\dfrac{\pi}{4}$

$=\sec^2\left(\dfrac{\pi}{4}\right)=(\sqrt{2})^2=2$

8. By *L'Hôpital's* Rule, $\lim\limits_{x\to\pi}\dfrac{e^x-e^\pi}{x^e-\pi^e}=\lim\limits_{x\to\pi}\dfrac{e^x}{ex^{e-1}}$
$=\lim\limits_{x\to\pi}\dfrac{e^{x-1}}{x^{e-1}}=\dfrac{e^{\pi-1}}{\pi^{e-1}}$.

Chapter 7

9. See Figure DS-2 on the next page.

10. I. Since the graph of g is decreasing and then increasing, it is not monotonic.

II. Since the graph of g is a smooth curve, g' is continuous.

III. Since the graph of g is concave upward, $g''>0$.
Thus, only statements II and III are true.