The Theory of Financial Decision Making

Asset Pricing

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MSc in Accounting and Finance

MSC in Finance

MSC in Finance with Banking

MSc in Finance with Risk Management

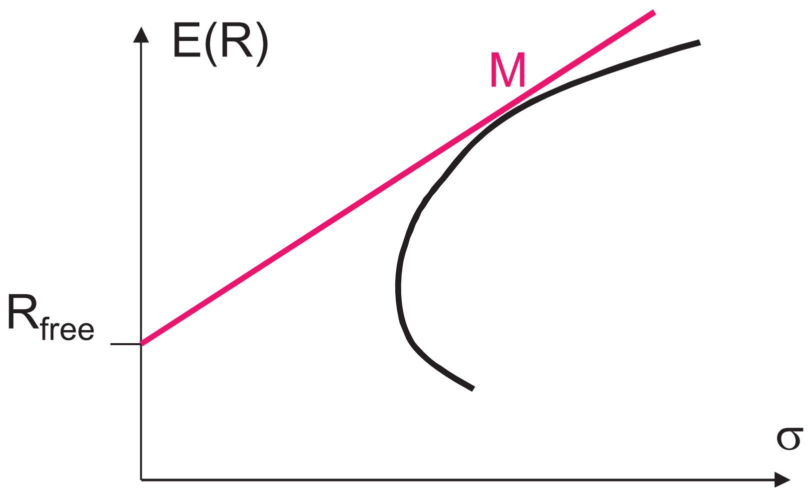
(week 5)

Literature

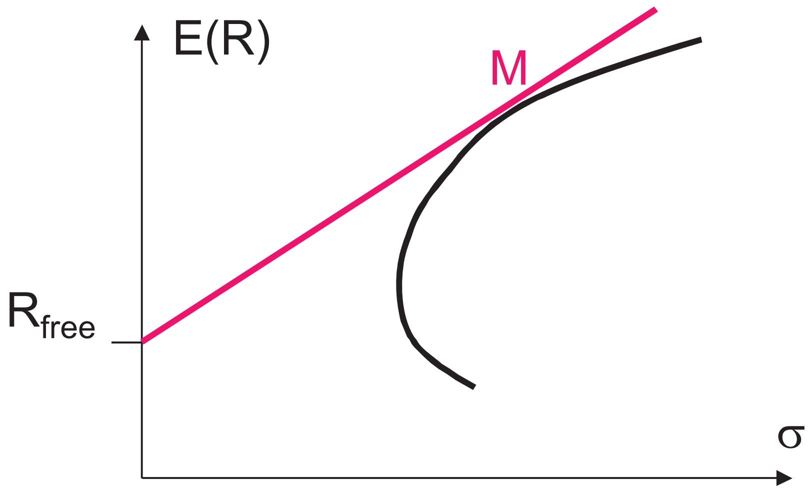
- CWS, chapter 6
- H.M. Markowitz, CAPM Investors do not get paid for bearing risk: a linear relation does not imply payment for risk, Journal of Portfolio Management, 2008 Winter, 90-94
- M. Anson, The Beta Continuum: From Classic Beta to Bulk Beta, Journal of Portfolio Management, 2008 Winter, 53-64
- P.A. Grout and A. Zalewska, The Impact of Regulation on Market Risk, Journal of Financial Economics, 2006(80), 149-184

Two-fund separation theorem

- Each utility-maximising investor's portfolio will be somewhere on the CML, i.e., he/she invests in a combination of
  - the risk-free asset
  - a portfolio of risky assets that is determined by the tangential line drawn from the risk-free rate of return to the efficient set of risky assets.



What will happen if you invest in the market portfolio and a risky asset?



$$E(R_p) = aE(R_i) + (1-a)E(R_M)$$

$$\sigma(R_p) = \sqrt{a^2\sigma_i^2 + (1-a)^2\sigma_M^2 + 2a(1-a)\sigma_{iM}}$$

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- When we start changing weights of the portfolio, i.e., how much we invest in the risky asset and in the market portfolio, the expected return and risk will change accordingly:

$$\frac{\partial E(R_p)}{\partial a} = E(R_i) - E(R_M)$$

$$\frac{\partial \sigma(R_p)}{\partial a} = \frac{1}{2}\frac{2a\sigma_i^2 - 2\sigma_M^2 + 2a\sigma_M^2 + 2\sigma_{iM} - 4a\sigma_{iM}}{\sqrt{a^2\sigma_i^2 + (1-a)^2\sigma_M^2 + 2a(1-a)\sigma_{iM}}}$$

But in equilibrium, the market portfolio contains all assets, so the proportion put in any risky asset above what is in the market portfolio must be zero, hence:

$$\left.\frac{\partial E(R_p)}{\partial a}\right|_{a=0} = E(R_i) - E(R_M)$$

$$\left.\frac{\partial \sigma(R_p)}{\partial a}\right|_{a=0} = \frac{1}{2}\frac{-2\sigma_M^2 + 2\sigma_{iM}}{\sqrt{\sigma_M^2}} = \frac{\sigma_{iM} - \sigma_M^2}{\sigma_M}$$

- The risk-return trade-off for the market portfolio is:

$$\left.\frac{\partial E(R_p)/\partial a}{\partial \sigma(R_p)/\partial a}\right|_{a=0} = \frac{E(R_i)-E(R_M)}{(\sigma_{iM}-\sigma_M^2)/\sigma_M}$$

but the slope must be the same as the slope of the capital market line $\frac{\mathrm{E}(\mathrm{R}_\mathrm{M})-\mathrm{R}_\mathrm{free}}{\sigma(\mathrm{R}_\mathrm{M})}$

so

$$\frac{E(R_i)-E(R_M)}{(\sigma_{iM}-\sigma_M^2)/\sigma_M} = \frac{E(R_M)-R_{free}}{\sigma_M}$$

$$\frac{E(R_i)-E(R_M)}{(\sigma_{iM}-\sigma_M^2)} = \frac{E(R_M)-R_{free}}{\sigma_M^2}$$

$$E(R_i)-E(R_M) = \left(\frac{\sigma_{iM}}{\sigma_M^2}-1\right)\left(E(R_M)-R_{free}\right)$$

$$E(R_i)-R_{free} = \frac{\sigma_{iM}}{\sigma_M^2}\left(E(R_M)-R_{free}\right)$$

CAPM: Capital Asset Pricing Model

$$E(R_i) - R_{free} = \beta\left(E(R_M) - R_{free}\right) \text{ where } \beta = \sigma_{iM} / \sigma_M^2$$

- Empirical representations:

$$R_i - R_{free} = \beta\left(R_M - R_{free}\right) + \varepsilon \quad \text{or} \quad R_i = a + \beta R_M + \varepsilon$$

- Total risk = systematic risk + nonsystematic risk

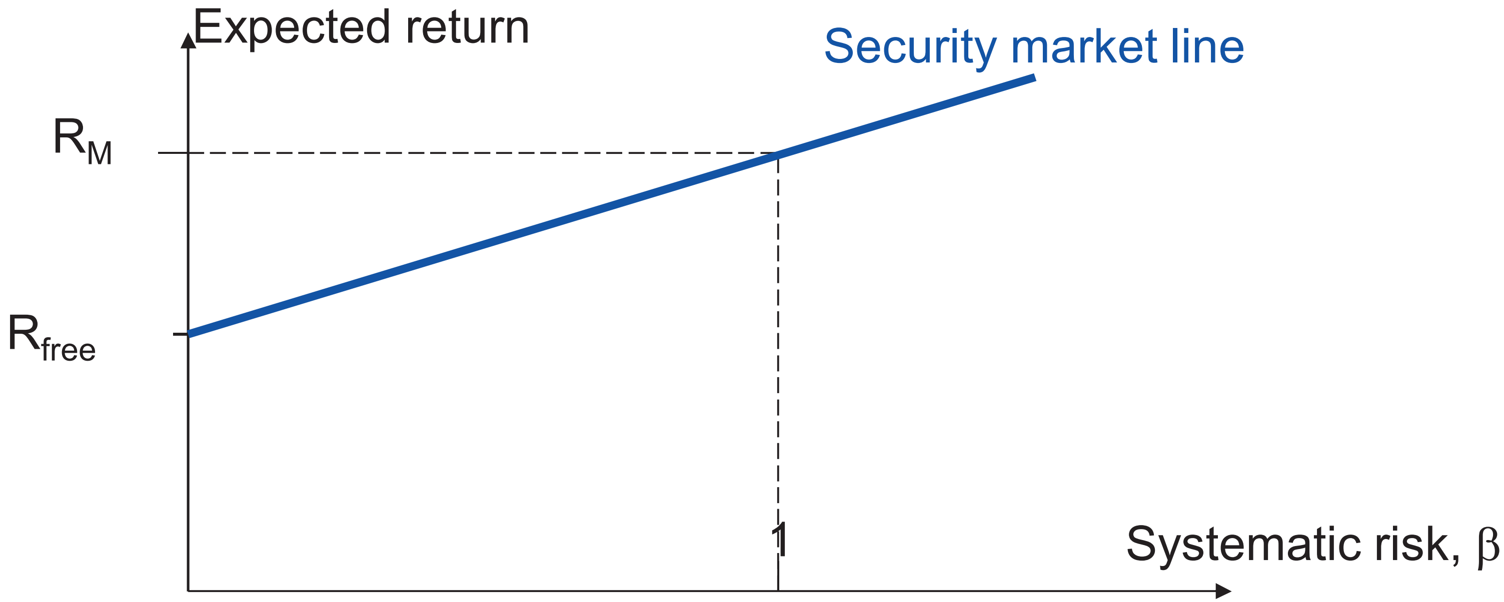
$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_\varepsilon^2$$

- Properties of the market portfolio:
  - Assets included: all securities
  - Weights $w_i = \dfrac{\text{market value of the individual asset}}{\text{market value of all assets}}$

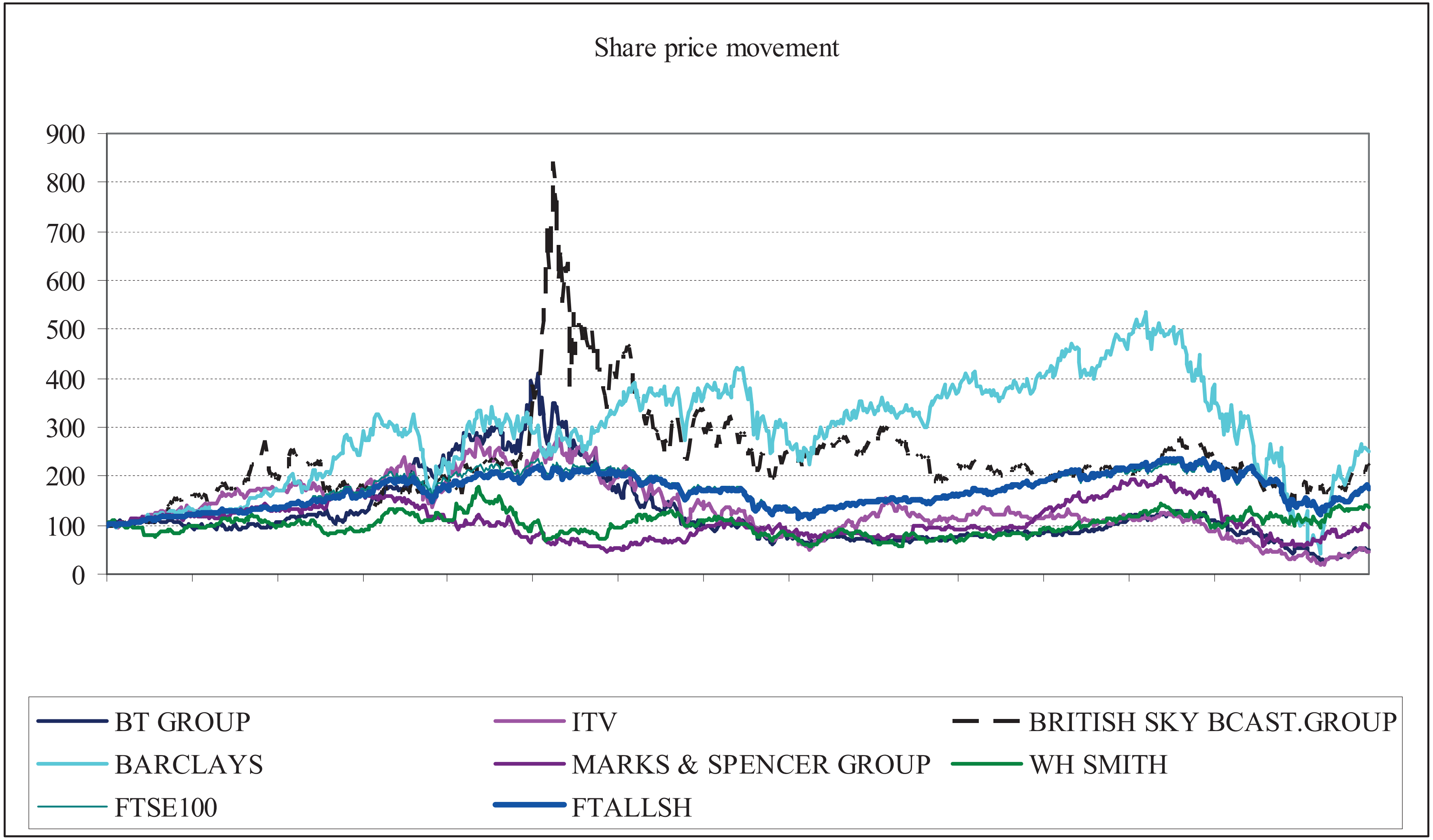
What is the β of the market portfolio?

$$CAPM: \quad E(R_i) - R_{free} = \beta\left(E(R_M) - R_{free}\right) \text{ where } \beta = \sigma_{iM} / \sigma_M^2$$

Hence, $\beta_M = 1$.



Example


Share price movement
900
800
700
600
500
400
300
200
100
0
BT GROUP
ITV
BRITISH SKY BCAST.GROUP
BARCLAYS
MARKS & SPENCER GROUP
WH SMITH
FTSE100
FTALLSH