2025年中考北师大版数学复习练案

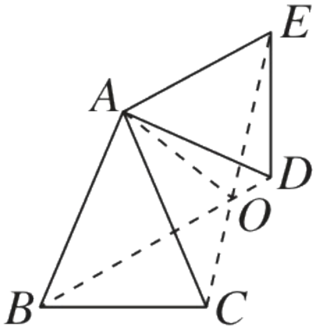
旋转中的模型

模型1 “手拉手”模型

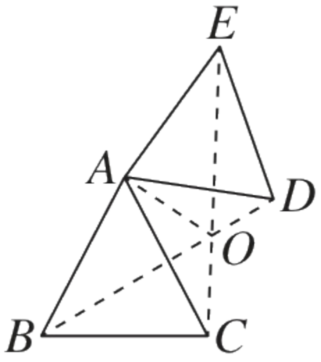
【模型解读】

条件：$AB=AC,AD=AE,\angle BAC=\angle DAE.$

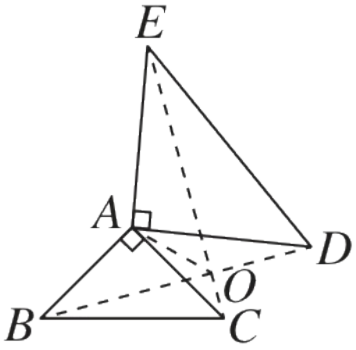
结论：①$\triangle ABD\cong\triangle ACE$;②$\angle BOC=\angle BAC$;③$AO$平分$\angle BOE$.



等腰三角形



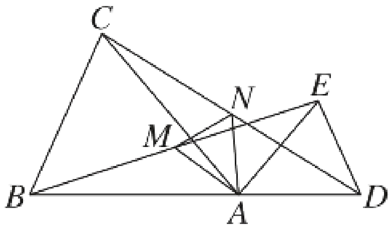
等边三角形



等腰直角三角形

a.双等腰三角形

1.如图①，在△ABC和△ADE中，$AB=AC$，$AD=AE$，$\angle BAC=\angle DAE$，且点B，A，D在一条直线上，连接BE，CD，M，N分别为BE，CD的中点.



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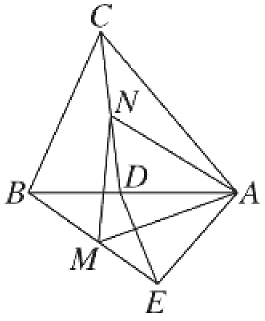

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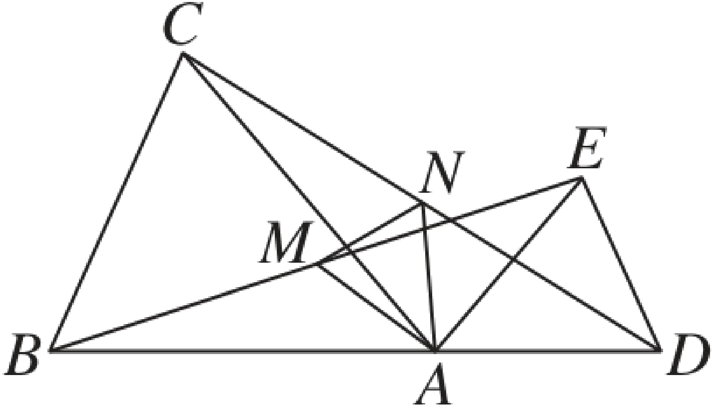
（1）求证：



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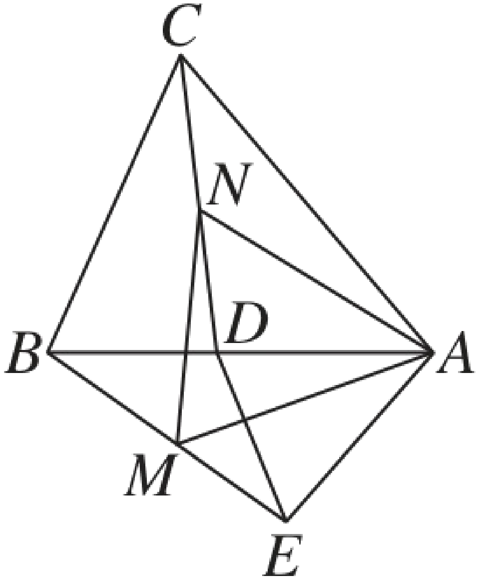


②

①$BE=CD$;

【证明】$\because \angle BAC=\angle DAE$，$\therefore \angle BAE=\angle CAD$.

又$\because AB=AC$，$AE=AD$，$\therefore \triangle ABE\cong\triangle ACD$.$\therefore BE=CD$.

②$\triangle AMN$是等腰三角形.

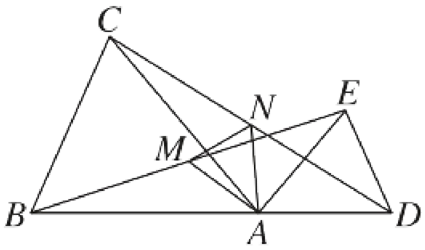
由$\triangle ABE\cong\triangle ACD$，得$\angle ABE=\angle ACD$，$BE=CD$.

$\because M$，N分别是BE，CD的中点，$\therefore BM=CN$.

又$\because AB=AC$，$\therefore \triangle ABM\cong\triangle ACN$.

$\therefore AM=AN$，$\therefore \triangle AMN$为等腰三角形.

（2）在图①的基础上，将$\triangle ADE$绕点A按顺时针方向旋转180°，其他条件不变，得到如图②所示的图形.（1）中的两个结论是否仍然成立?说明理由.



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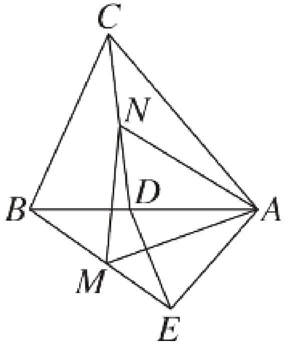

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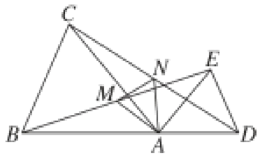
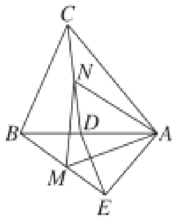
【解】（1）中的两个结论仍然成立.





理由如下：$\because BA = CA$,

$\angle BAE = \angle CAD, AE = AD$,

$\therefore \triangle ABE \cong \triangle ACD$(SAS).

$\therefore BE = CD$，$\angle ABE = \angle ACD$，即$\angle ABM = \angle ACN$.

$\because M, N$分别为BE, CD的中点，$\therefore BM = CN$.

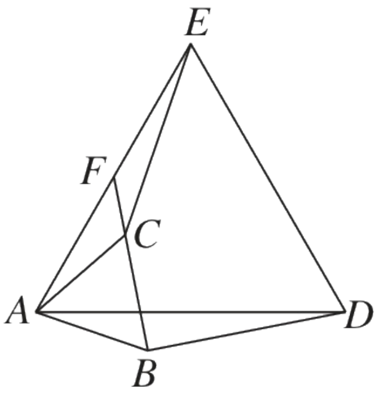
又$\because AB = AC, \therefore \triangle ABM \cong \triangle ACN$(SAS).

$\therefore AN = AM. \therefore \triangle AMN$是等腰三角形.

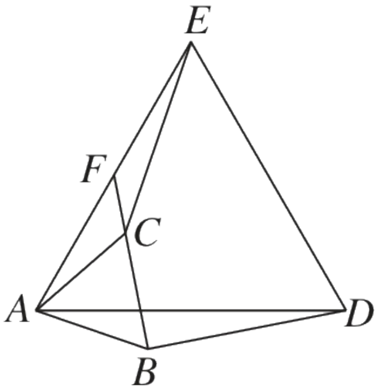
b.双等边三角形

2.如图，$\triangle ABC$与$\triangle ADE$均为等边三角形，$\angle CBD = 90°$，

$BD = \sqrt{3}AB$,连接CE,延长BC交AE于点F.



（1）求$\angle FCE$的度数；



【解】$\because \triangle ABC$与$\triangle ADE$均为等边三角形，

$\therefore AD = AE, AB = AC,$

$\angle BAC = \angle DAE = \angle ABC = \angle ACB = 60^\circ$.

$\therefore \angle DAB = \angle EAC. \therefore \triangle DAB \cong \triangle EAC.$

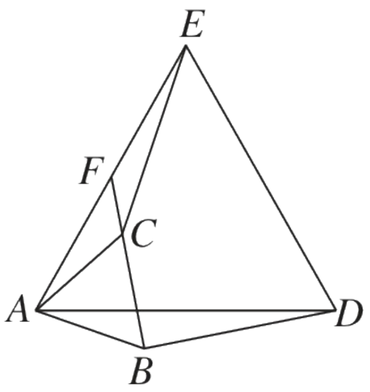
$\therefore \angle ABD = \angle ACE.$

又$\because \angle ABC = 60^\circ$,$\angle CBD = 90^\circ$,

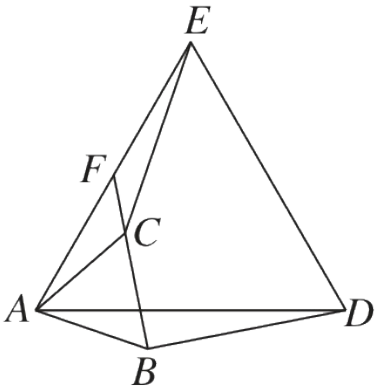
$\therefore \angle ACE = \angle ABD = 60^\circ + 90^\circ = 150^\circ$.

$\because \angle ACB = 60^\circ$,

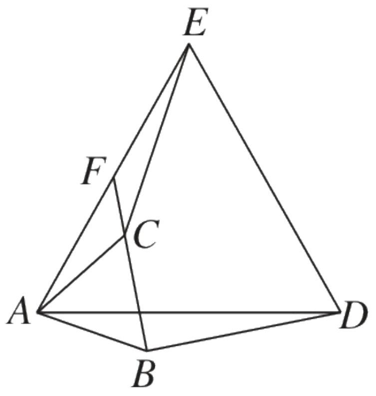
$\therefore \angle ACF = 120^\circ$.$\therefore \angle FCE = 30^\circ$.



（2）求证：$AF = EF$.



【证明】过点A作$AN \perp BC$于点N，过点E作$EM \perp BF$,交BF的延长线于点M.



设$AB = AC = BC = x$,则$BD = \sqrt{3}x$,

$BN = CN = \frac{1}{2}x$.

$\therefore AN = \sqrt{AB^2 - BN^2} = \frac{\sqrt{3}}{2}x$.

由（1）得$\triangle DAB \cong \triangle EAC$,$\therefore BD = CE = \sqrt{3}x$.

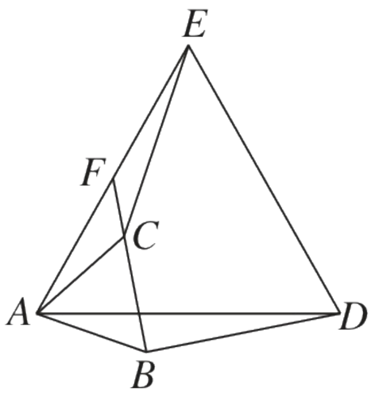
$\because \angle CME = 90^\circ$,$\angle FCE = 30^\circ$,

$\therefore ME = \frac{1}{2}CE = \frac{\sqrt{3}}{2}x.$

$\therefore ME = AN.$

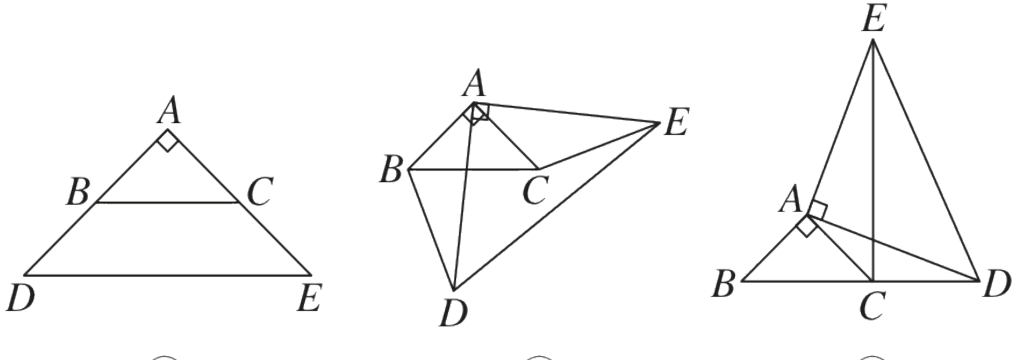
又$\because \angle ANF = \angle EMF = 90^\circ$,$\angle AFN = \angle EFM$,

$\therefore \triangle AFN \cong \triangle EFM. \therefore AF = EF.$



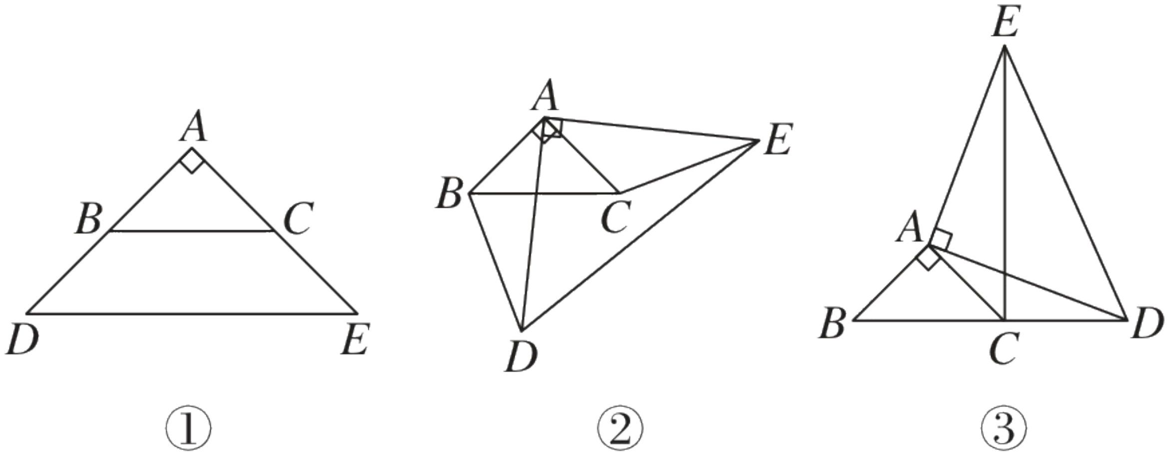
c.双等腰直角三角形

3.[2024盘锦兴隆台区模拟] 【感知】如图①，$\triangle ABC$和$\triangle ADE$都是等腰直角三角形，$\angle BAC=\angle DAE=90^{\circ}$，点$B$在线段$AD$上，点$C$在线段$AE$上，我们很容易得到$BD=CE$.



① ② ③

【探究】 将$\triangle ADE$绕点A逆时针旋转$\beta(0° < \beta < 90°)$，如图②，连接BD和CE，此时$BD = CE$是否依然成立？若成立，写出证明过程;若不成立，说明理由.



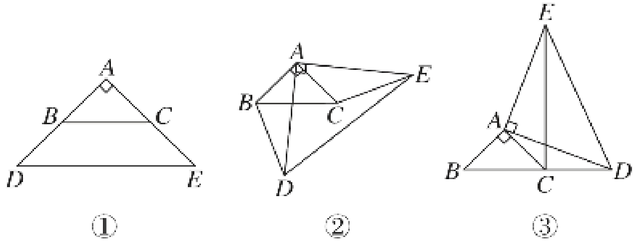
【解】$BD = CE$依然成立.证明如下:

$\because \triangle ABC$和$\triangle ADE$都是等腰直角三角形,

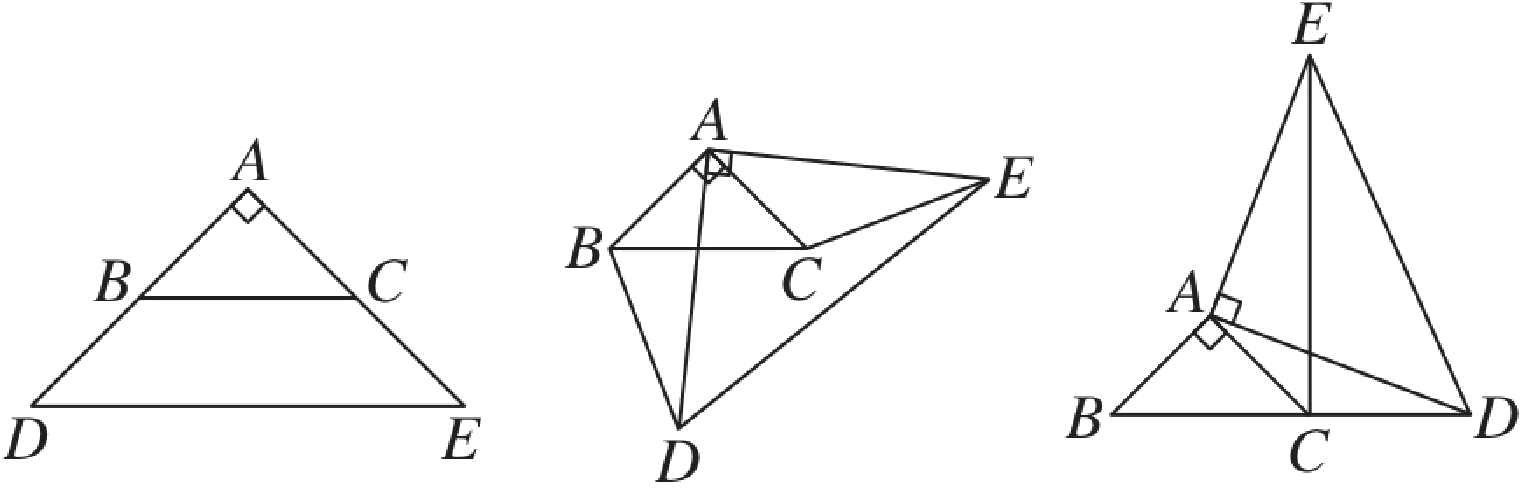
$\angle BAC = \angle DAE = 90^\circ$,

$\therefore AB = AC$, $AD = AE, \angle BAD = \angle CAE$.

$\therefore \triangle ABD \cong \triangle ACE$(SAS). $\therefore BD = CE$.



【应用】 如图③，当△ADE绕点A逆时针旋转，使得点D落在BC的延长线上时，连接CE.



① ② ③

①$\angle ACE$的度数是__45°__；

②若$AB = AC = 2\sqrt{2}, CD = 4$，求线段$DE$的长.

$\because AB = AC = 2\sqrt{2}$，$\therefore BC = \sqrt{AB^2 + AC^2} = 4$.

易知$\triangle ACE \cong \triangle ABD$，$\therefore CE = BD = BC + CD = 4 + 4 = 8$.

易知$\angle BCE = 90°$，$\therefore \angle ECD = 90°$.

$\therefore DE = \sqrt{CE^2 + CD^2} = \sqrt{8^2 + 4^2} = 4\sqrt{5}$.

模型2 “半角”模型

a.90°角夹45°角

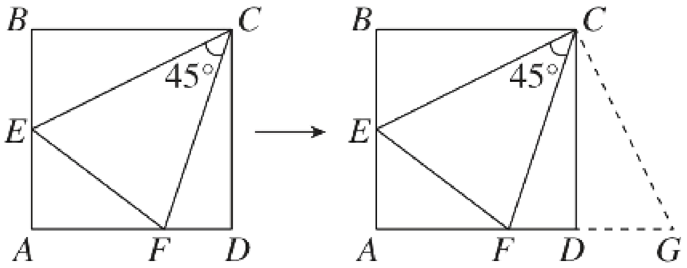
【模型解读】（1）**正方形半角模型**

条件：如图①，四边形$ABCD$是正方形，$\angle ECF=45°$．

结论：①$\triangle BCE\cong\triangle DCG$；②$\triangle CEF\cong\triangle CGF$；

③$EF=BE+DF$；④$\triangle AEF$的周长$=2AB$；

⑤EC,FC分别平分$\angle BEF$和$\angle EFD$.



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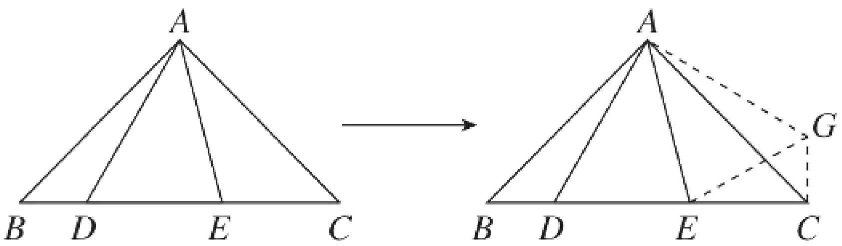

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（2）**等腰直角三角形半角模型**

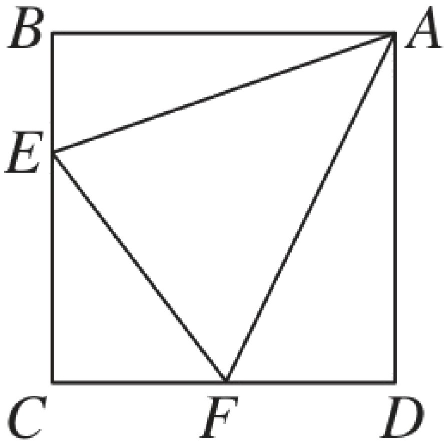
条件：如图②，$\triangle ABC$是等腰直角三角形，$\angle DAE = 45°$.

结论：①$\triangle BAD \cong \triangle CAG$；②$\triangle DAE \cong \triangle GAE$；

③$\angle ECG = 90°$ ；④$DE^2 = BD^2 + EC^2$.

4.【问题背景】

（1）如图①，点E,F分别在正方形$ABCD$的边BC,CD上，$\angle EAF=45^{\circ}$，连接EF，则有$EF=BE+DF$，试说明理由；



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【解】∵ 四边形$ABCD$是正方形,

∴ $AB = AD$，$\angle B = \angle BAD = 90°$.

把$\triangle ABE$绕点A逆时针旋转90° 得到$\triangle ADG$，如图①,

则$\angle DAG = \angle BAE$，$AE = AG$，$BE = DG$,$\angle ADG = \angle B = 90°$

∴ $\angle FDG = 180°$.

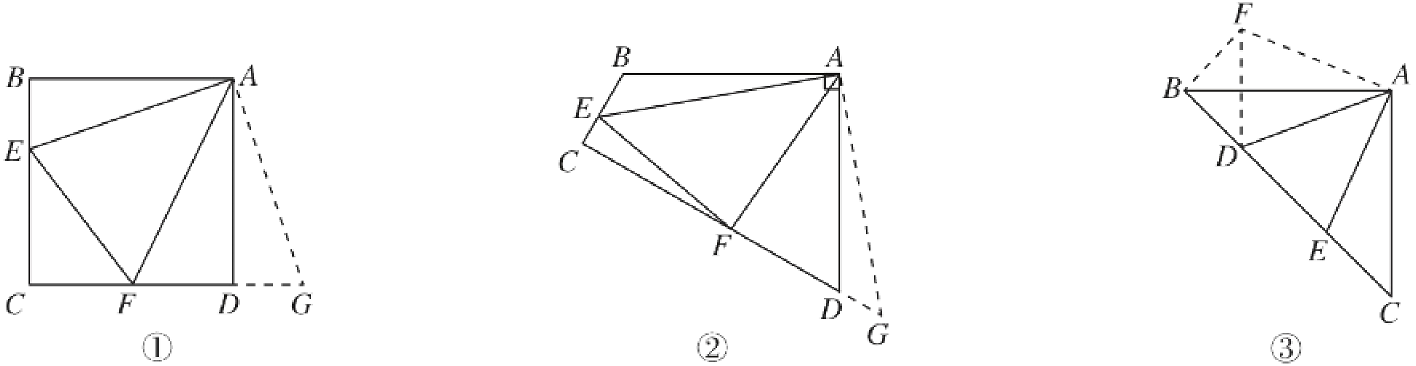
∴ 点F，D,G共线.

$\because \angle EAF = 45°$ $, \therefore \angle FAG = \angle FAD + \angle GAD = \angle FAD + \angle BAE = 90° - \angle EAF = 45°$.

$\therefore \angle EAF = \angle FAG$.

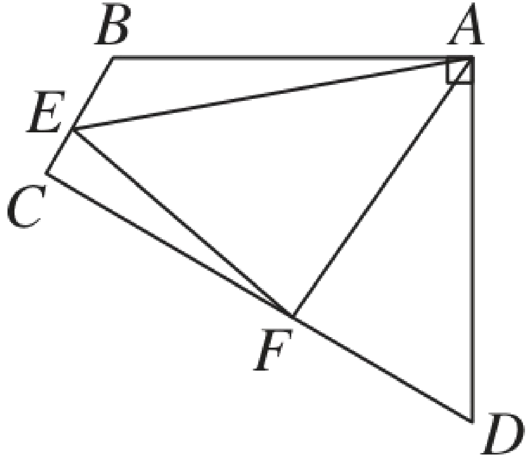
又$\because AF = AF, \therefore \triangle AGF \cong \triangle AEF$(SAS).

$\therefore EF = FG = DG + DF = BE + DF$.



【迁移应用】

（2）如图②，四边形$ABCD$中，$AB=AD$，$\angle BAD=90°$，点E,F分别在边BC,CD上，$\angle EAF=45°$，若$\angle B$，$\angle D$都不是直角，且$\angle B+\angle D=180°$，试探究EF,BE,DF之间的数量关系；



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∵ $AB=AD$，$\angle BAD=90°$，

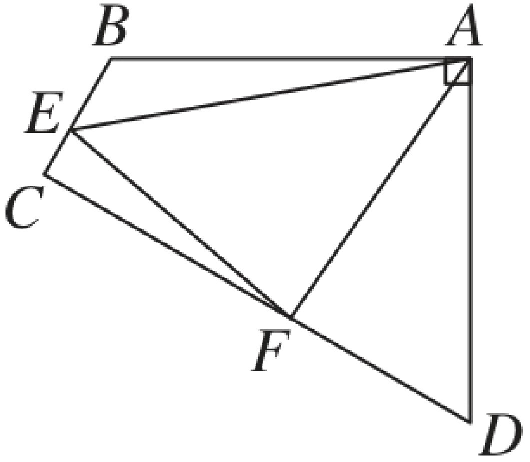
∴ 把$\triangle ABE$绕点A逆时针旋转90°得到$\triangle ADG$，如图②.

∴ $\angle BAE=\angle DAG$，$\angle B=\angle ADG$，

$AE=AG$，$BE=DG$.

∵ $\angle ADC+\angle B=180°$，

∴ $\angle ADC+\angle ADG=180°$，即$\angle FDG=180°$.



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∴ 点F,D,G共线.

∵ $\angle BAD=90°$ ， $\angle EAF=45°$ ，

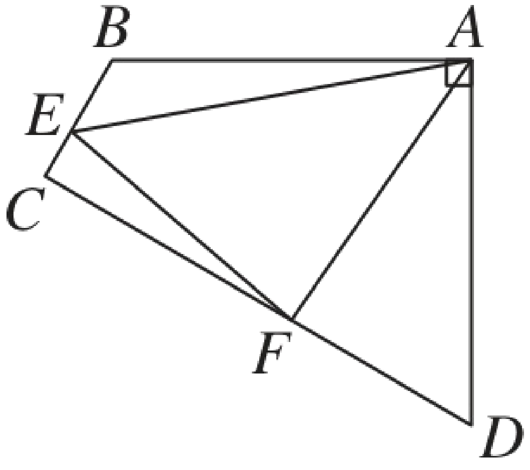
∴ $\angle BAE+\angle DAF=45°$.

∴ $\angle FAG=45°$.∴ $\angle EAF=\angle FAG$.

又∵ $AF=AF$,

∴ $\triangle AFE\cong\triangle AFG$(SAS).

∴ $EF=FG=DG+FD=BE+DF$.



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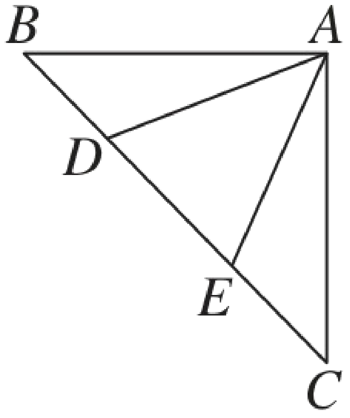
【联系拓展】

（3）如图③，在$\triangle ABC$中，$\angle BAC=90^{\circ}$，$AB=AC$，点D,E均在边BC上，且$\angle DAE=45^{\circ}$，猜想BD,DE,EC满足的等量关系（直接写出结论，不需要证明）．

$BD^2+CE^2=DE^2$.



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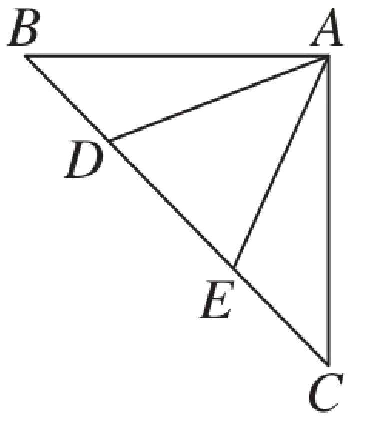
【点拨】如图③,把$\triangle ACE$绕点A顺时针旋转90°得到$\triangle ABF$，连接DF，则



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$\angle FAB=\angle CAE$，$BF=CE,AF=AE$，

$\angle ABF=\angle C$.

$\because \angle BAC=90°$，$\angle DAE=45°$，

$\therefore \angle BAD+\angle CAE=45°$.

$\therefore \angle FAD=\angle BAD+\angle BAF=45°$.

$\therefore \angle FAD = \angle DAE$.

又$\because AD = AD$,

$\therefore \triangle ADF \cong \triangle ADE$(SAS).$\therefore DF = DE$.

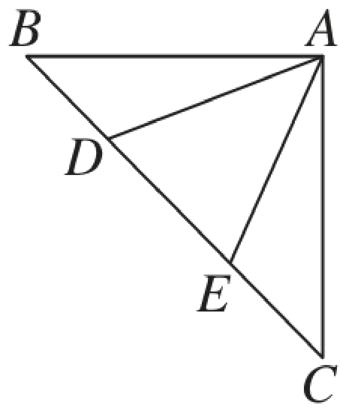
$\because \angle BAC = 90°$,$\therefore$ 易得

$\angle ABF + \angle ABC = \angle C + \angle ABC = 90°$,即

$\angle FBD = 90°$.

$\therefore BD^2 + BF^2 = DF^2$.

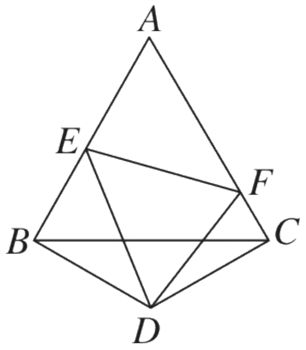
$\therefore BD^2 + CE^2 = DE^2$.

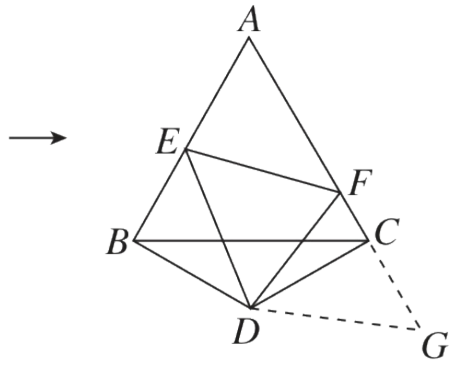


③

b.120° 角夹60° 角

【模型解读】条件：如图，$\triangle ABC$是等边三角形，$\triangle BDC$是等腰三角形，且$BD=CD$，$\angle BDC=120°$，$\angle EDF=60°$.





结论：①$\triangle BDE\cong\triangle CDG$；②$\triangle EDF\cong\triangle GDF$；③$EF=BE+FC$；④$\triangle AEF$的周长$=2AB$；⑤$ED,FD$分别平分$\angle BEF$和$\angle EFC$.

5.【问题情境】

在等边三角形ABC的两边AB，AC上分别有两点M，N，点D为$\triangle ABC$外一点，且$\angle MDN = 60^\circ$，$\angle BDC = 120^\circ$，$BD = DC$.