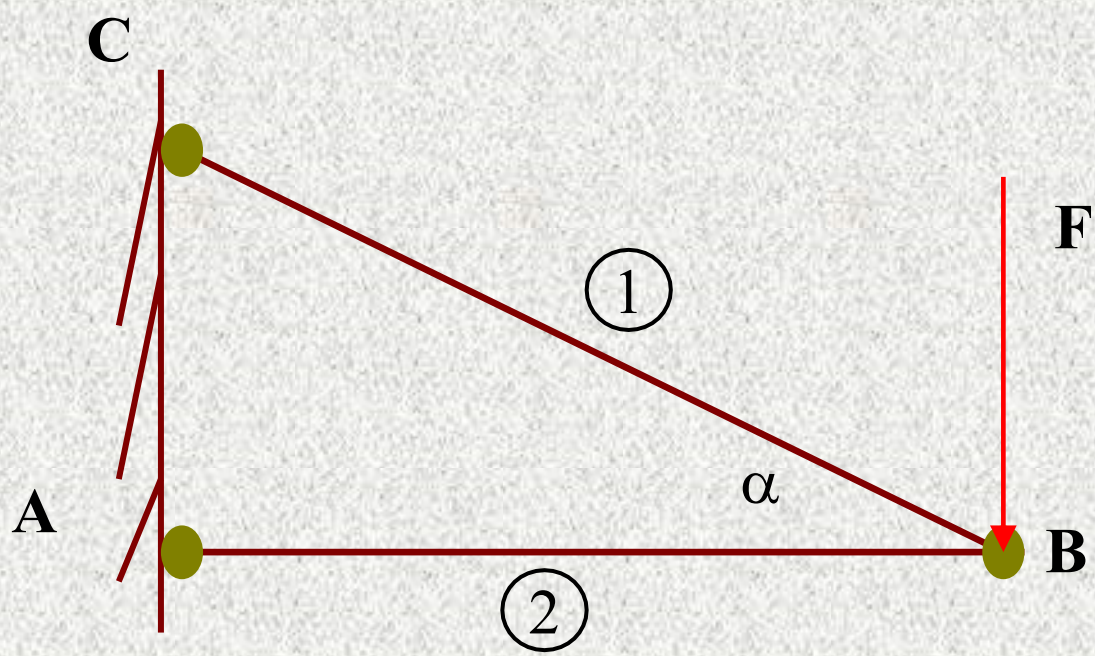
Mechanics of Materials

Chapter 10
Internal Reactions: Stress for Axial Loads

§10-5 Shear Stress

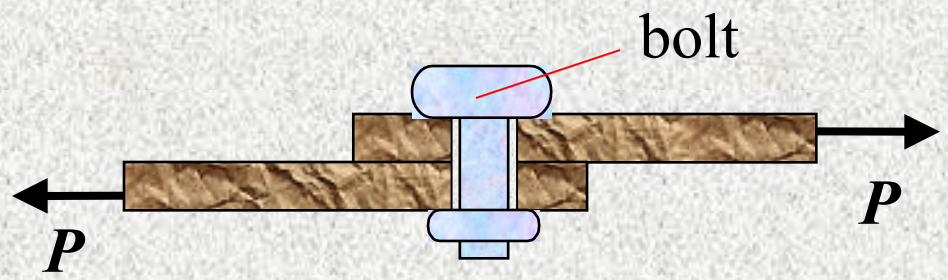


Production of the shearing stress

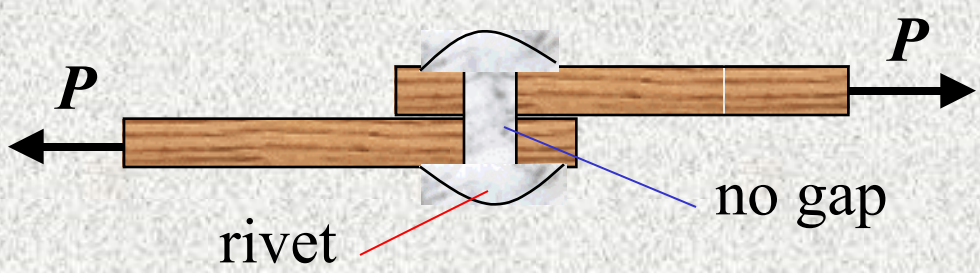
1、Characteristics of loads and deformation of connecting members:

1)Connecting member

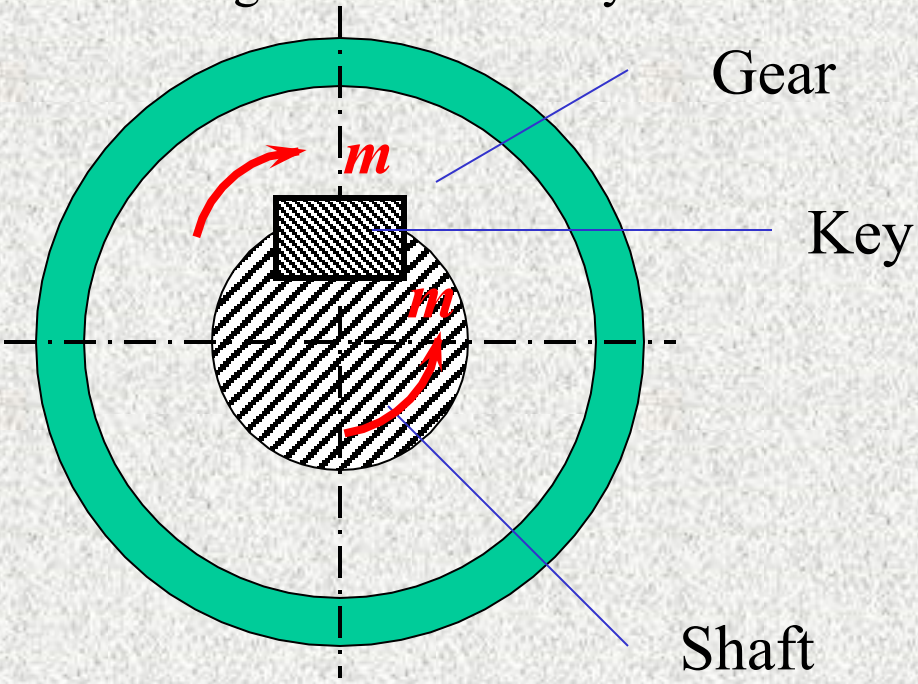
The structure member that connects one member to another is called the **connecting member**. Such as：bolts、rivets、keys etc. The connecting member is small，but it plays the role of passing loads.

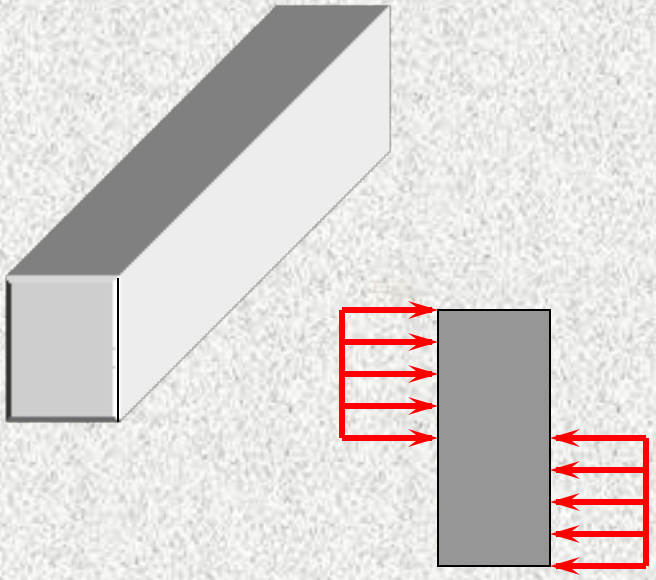


Characteristic： It can pass general loads and can be dismounted.



Characteristic: It can pass general loads, but can not be dismounted.for example, the truss in a bridge is connected by it.

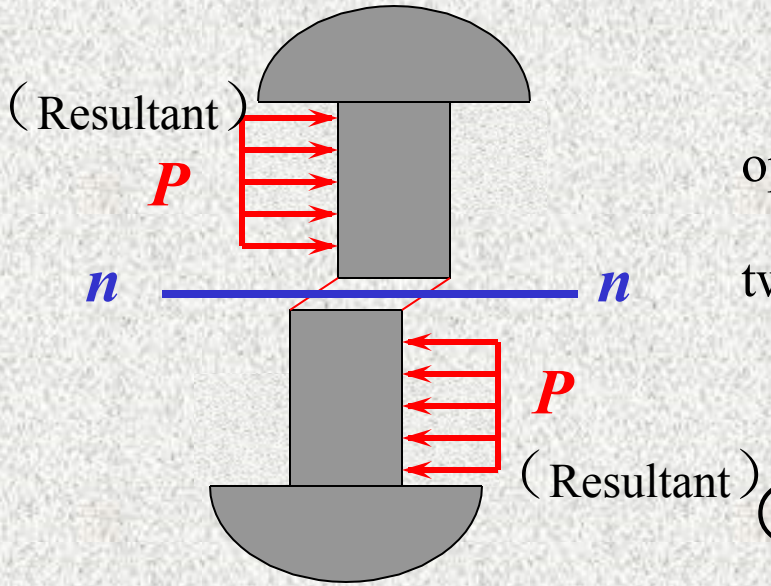




Characteristic: It can pass torques.

2) Characteristics of loads and deformation:

Use a rivet as an example:

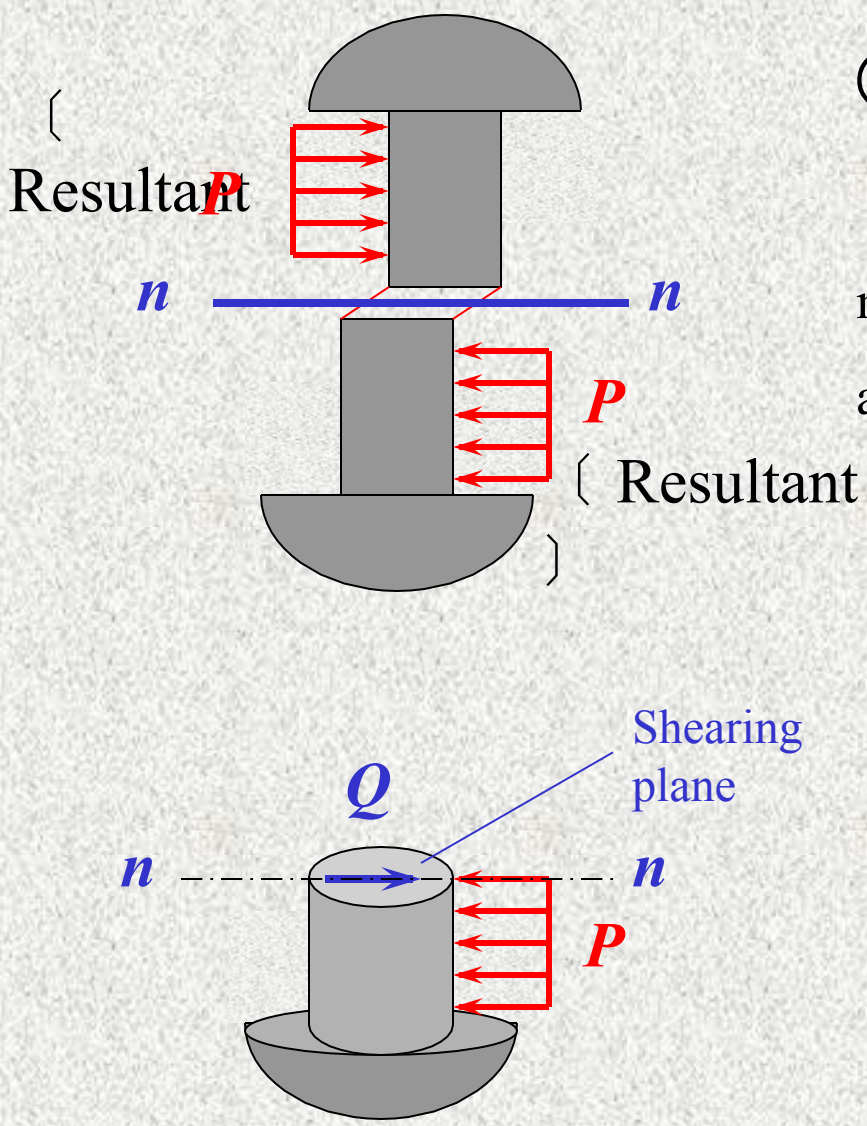


① Characteristic of loads:

The rivet is subjected to two equal and opposite forces. The acting lines of these two forces are very close.

② Characteristic of deformation:

Two parts subjected to two equal and opposite forces tend to shift over one another along the junction plane of two forces.

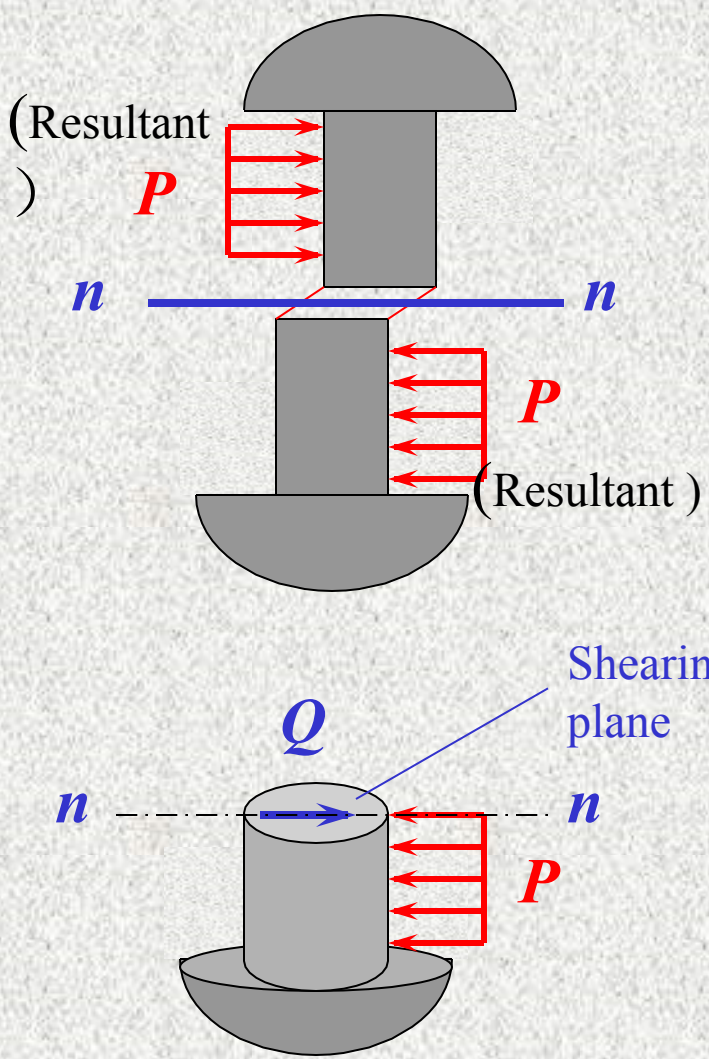


③Shearing plane:

The plane along which two parts of the member tend to shift over one another. Such as ***n– n*** .

④Internal force on shearing plane:

Internal force — Shearing force ***Q*** , its acting line is parallel to the shearing plane.



3)Three kinds of breakage at joint:

①Failure due to shear

Snip along the shearing plane of the rivet, such as along section ***n– n.***

②Breakage due to bearing

Fail due to mutual bearing between the rivet and the steel plate in their connecting plane.

③Breakage due to tension

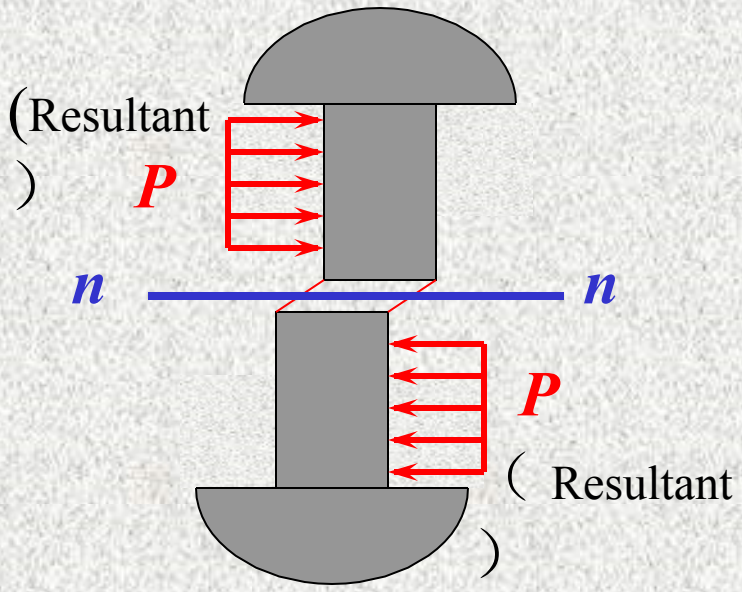
The steel plate is weakened in the section in which the rivet holes exist and stress in the weakened section increases so that the steel plate is easily broken due to tension at the connecting position.

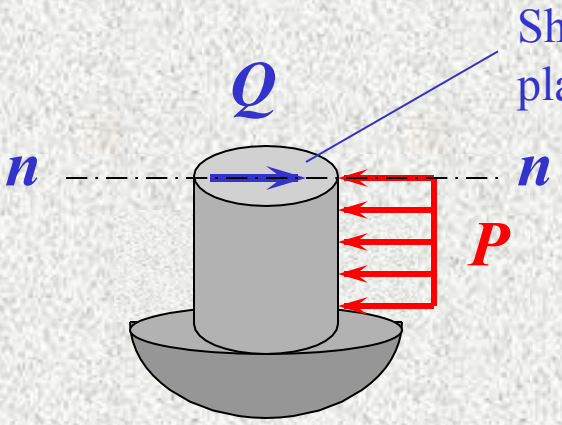
2、Practical calculation of shear

Method of the practical calculation: According to possibility of breakage of the member some assumptions by which basic characteristic subjected to force actions can be reflected and calculations can be simplified are used. Then calculate its nominal stress, determine the corresponding permissible stress in accordance with the result of direct test. At last do the strength calculation.

Applying range: volume of the member is not large and real stress is quite complex. Such as the connecting pieces etc.

Assumption of practical calculation: Assume that shearing stress is distributed uniformly in the shearing plane and equal to the average shearing stress.





1) Shearing plane--A_Q: Shifting plane.

Shearing force--Q: Internal force on the shearing plane.

2) Nominal shearing force--τ:

$$\tau = \frac{Q}{A_Q}$$

3) Strength condition of shear:

$$\tau = \frac{Q}{A_Q} \le [\tau], \text{ where } [\tau] = \frac{\tau_{jx}}{n}$$

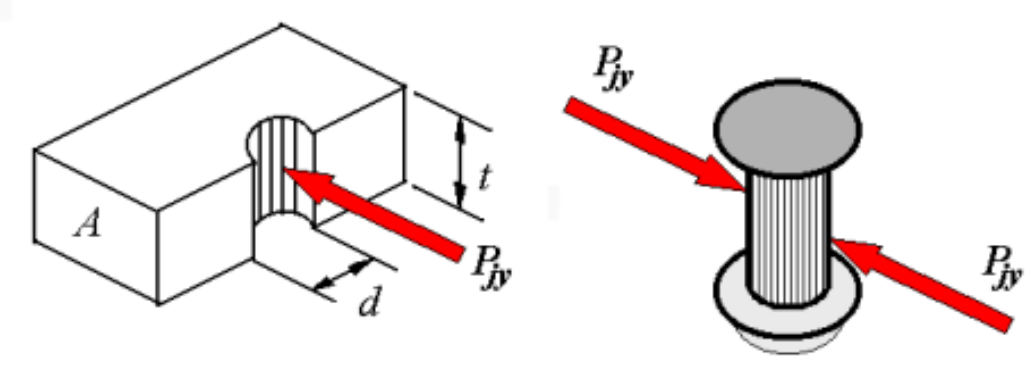
Working stress should not exceed the permissible stress.

§10-6 Bearing Stress

Bearing： The phenomenon that there is pressure on the partial area of a member.

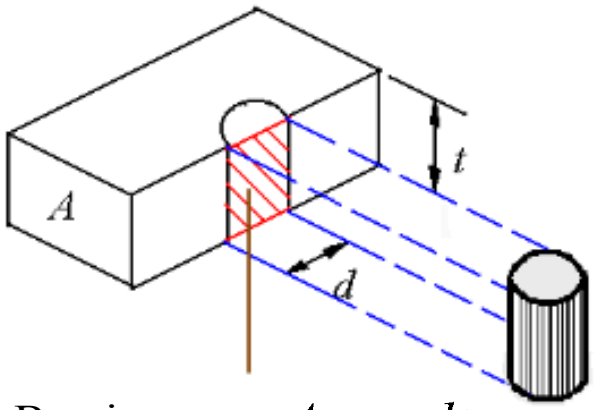
Bearing force： The resultant force acting on the bearing plane， designated by P_{jy}.

1）Bearing force—P_{jy} ： The resultant force acting on the touching plane.



Assumption： Bearing stresses are distributed uniformly over the effective bearing plane.

2) Bearing area: Area of the projection plane of the touching plane in the direction perpendicular to P_{jy}



Bearing area $A_{jy} = dt$

3) Strength condition of bearing:

Working bearing stress should not exceed the permissible bearing stress

$$\sigma_{jy} = \frac{P_{jy}}{A_{jy}} \leq [\sigma_{jy}]$$

4、Applications

1)**Check the strength** $\tau \le [\tau]$; $\sigma_{jy} \le [\sigma_{jy}]$

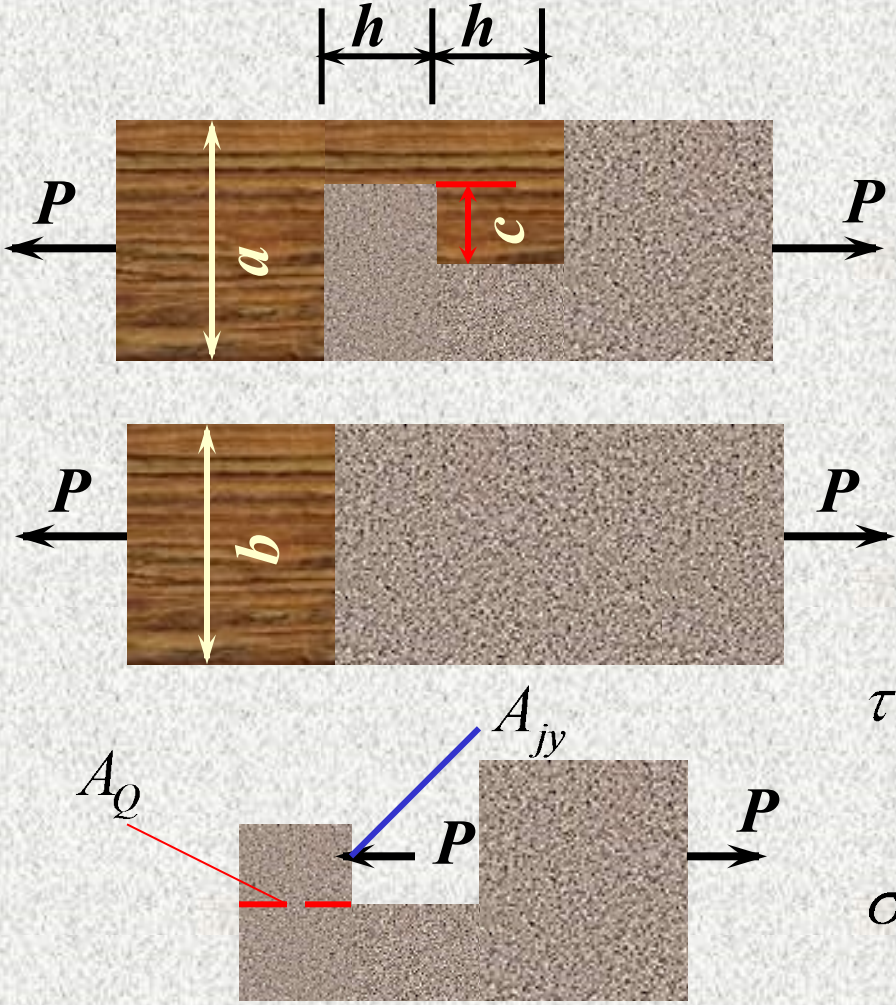
2)**Select the cross section area** $A_Q \ge \dfrac{Q}{[\tau]}$; $A_{jy} \ge \dfrac{P_{jy}}{[\sigma_{jy}]}$

3)**Determine the external load**

$$Q \le A_Q[\tau];$$

$$P_{jy} \le A_{jy}[\sigma_{jy}]$$

Example 1 A wooden tenon joint is shown in the figure. Knowing that the quantities are $a = b$ =12cm，h=35cm，c=4.5cm and P=40KN. Try to determine the shearing stress and bearing stress for the joint.



Solution: ①: Free body diagram is shown in the figure

Shearing force is

$$A_Q = bh;\ Q = P$$

Bearing force is

$$A_{jy} = cb;\ P_{jy} = P$$

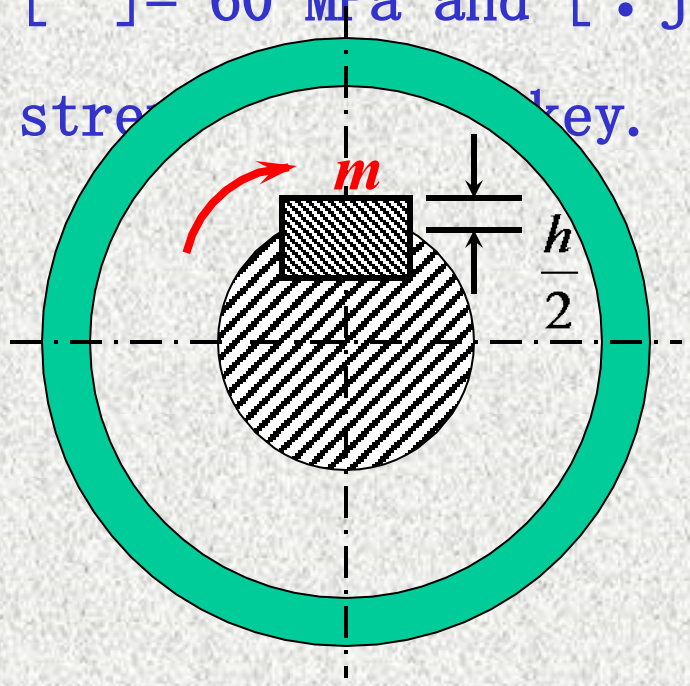
②: Shearing stress and bearing stress

$$\tau = \frac{Q}{A_Q} = \frac{P}{bh} = \frac{40}{12\times 35}\times 10^7 = 0.952\text{MPa}$$

$$\sigma_{jy} = \frac{P_{jy}}{A_{jy}} = \frac{P}{cb} = \frac{40}{4.5\times 12}\times 10^7 = 7.4\text{MPa}$$

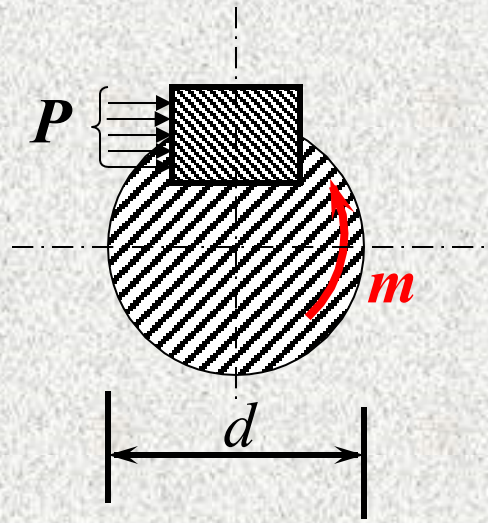
b×h×L=20 ×12 ×100). The torque that the key n

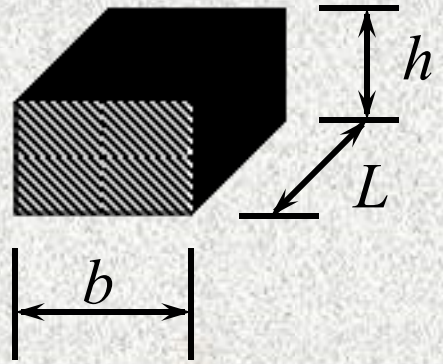
transmit is m=2KNm. Knowing the diameter of the shaft is d=70mm, the permissible shearing stress and the permissible bearing stress of the key are respectively [▲]= 60 MPa and [? jy]= 100 MPa. Try to check the stre key.



Solution: ①Free body diagram of the key is shown in the figure

$$P=\frac{2m}{d}=\frac{2\times 2}{0.07}=57\text{kN}$$

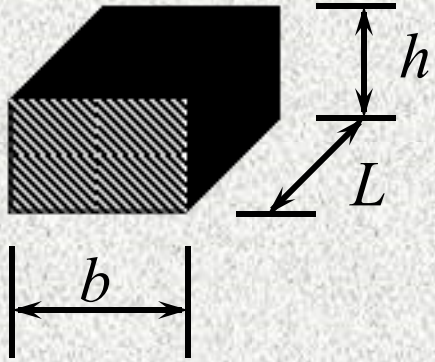


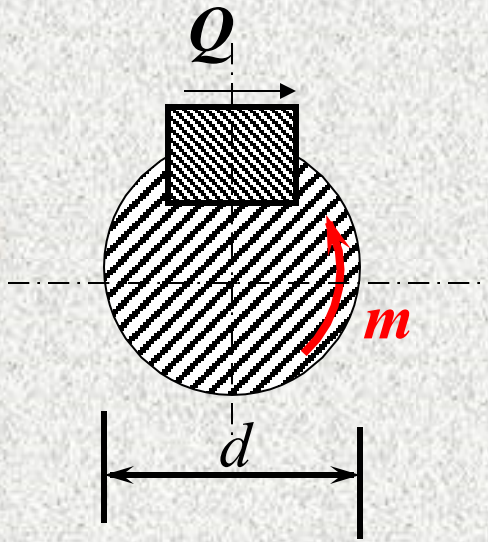


②Check the strength of shear and bearing $Q = P_{jy} = P$

$$\tau = \frac{Q}{A_Q} = \frac{P}{bL} = \frac{57 \times 10^3}{20 \times 100} = 28.6\text{MPa} \le [\tau]$$

$$\sigma_{jy} = \frac{P_{jy}}{A_{jy}} = \frac{P}{Lh/2} = \frac{57 \times 10^3}{100 \times 6} = 95.3\text{MPa} \le [\sigma_{jy}]$$





According to the above calculation, strength conditions of the key are satisfied.