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The thesis discusses the calculation and implementation of implied correlation as a valuation method for the implied volatility of a basket or index option implied volatility. We suggested how a trader may find a practical application for this useful and relatively simple tool.

We believe that implied correlation for dispersion trading is an extremely useful and potentially profitable vehicle for tracing any disparity/inefficiency in the market.

More importantly, we have introduced the concept of using Variance swaps in the execution of correlation dispersion trades. This is in contrast the more traditional method of trading strangles on index and basket options. Variance swaps require no delta hedging and are therefore less operationally intensive than standard delta-hedged option strategies. This method of trading should not be underestimated and receive merits for its attractive qualities.

Furthermore, we were able to show how Bollinger bands can be used as an indicator to identify potential trading opportunities. One anticipates that the dispersion strategy outlined has been designed to generate a consistent return with a low correlation to the underlying market in a variety of market regimes.

This thesis indicates that using Variance swaps in correlation dispersion trading strategies can be profitable, but a backtest on a larger number of trades is recommended.

INTRODUCTION

Typically implied correlation is used in dispersion trading to identify opportunities in which distortion / mispricing of implied volatility has created an identifiable (tradable) gap between the implied volatility of a benchmark index and its constituent members. The thesis will explain how this gap can be identified and how to trade dispersion. Implied correlation also enables better timing for implied volatility investing, as the model provides an indication of relative value of the constituent members of a basket to its benchmark index.

Dispersion trading has a number of theoretical and practical appeals. While a strict arbitrage lower bound exists, minimal variance hedging can be used to construct tracking baskets of options to capitalise on the mispricing of the two different volatility markets that arises from time to time.

Dispersion trading typically consists of a strategy where one simultaneously sells index options and buys options on the index components or, conversely, buys index options and sells options on the underlying stocks. The fundamental theme associated with dispersion trading is correlation trading.

The value of an index option is determined by the implied volatility of the constituent single stocks and their cross correlation coefficient. Hence, it is possible to work backwards to derive how correlated the market estimates the basket of stocks to be. The value of a single stock option does not take correlation into account as the option is on a single security. If correlation is less than 100% there is a spread between the index option implied volatility and the basket implied volatility. In order to calculate index implied volatility from single stock implied volatility we need to ascertain what level of cross correlation exists between the market weighted single stocks.

A dispersion trader relies upon the fact that when implied correlation peaks, index implied volatility is considered to be too high, the trader therefore enters into a trade whereby he/she sells index implied volatility and simultaneously buys a basket of single stock implied volatility (market weighted) to hedge his/her short exposure.
In the event that implied correlation troughs the opposite trade is entered in to with a short position in a basket of market weighted single stock implied volatilities and a simultaneous long position in index implied volatility as it is considered to be too cheap.

A typical dispersion relative-value position consists of buying single stock volatility, while selling index volatility. Since the index is composed of single stocks, the two are closely correlated. The standard dispersion position consists of buying equity at-the-money strangles, and selling index at-the-money strangles. The total position should be delta-hedged continuously to achieve the returns expected from arbitrage. However, traditional dispersion (through calls, puts or strangles) has its failings. Firstly, daily delta hedging entails replication errors and transaction costs. Furthermore, the strategy is path-dependent; depending on the market's evolution, the position can become vega-biased (vega-positive/negative), and develop into a non-hedged volatility position, rather than a play on pure stock correlation which is the intention of the arbitrage.

A cleaner way to address these shortcomings is to build a position of variance swaps. These instruments allow traders to effectively build a long/short position that better matches the desired arbitrage. Unlike the option strategy, the instruments are delta-neutral at all times and continuous trading is therefore redundant. Variance swaps are a product that investors have, more recently, been taking advantage of to fine-tune their risk profile and reduce peripheral risks, such as path-dependency.

The Variance swap market has grown steadily in recent years. The development has primarily been driven by investor demand to take direct volatility exposure without the cost and complexity of managing and delta-hedging a vanilla options position. Measuring the size of an over-the-counter market is, at the best of times difficult, but the market turnover size has been estimated, measured in options notional equivalent, to be in the proximity of €20 billion a year. To put this into context, the turnover of listed Eurostoxx 50 options is about €200 billion a year.
Variance swap liquidity initially developed on equity index underlyings, and more recently the single-stock variance swap market has begun to open up.

"….as the market for variance swaps has matured, there have been more trades, at narrower spreads, and with the ability to execute very large transactions. Users are mainly hedge funds, with occasional insurance companies, endowments and sophisticated pension funds executing trades…." (Dean Curnutt, head of equity derivatives strategy at Bank of America Securities).

In the process of its development, the market has attracted dispersion traders who buy and sell index variance swaps against single-stock variance swaps to take correlation-like exposures.

This thesis will attempt to illustrate how the calculation and implementation of implied correlation can be used in volatility-dispersion trading strategies. More importantly, we introduce the concept of using Variance swaps in the execution of correlation dispersion trades.

CHAPTER 1

Introducing dispersion trading

1.1 Concept of dispersion trading

Volatility dispersion trading is a popular hedged strategy designed to take advantage of relative value differences in implied volatilities between an index and a basket of component stocks, looking for a high degree of dispersion. This strategy typically involves short option positions on an index, against which long option positions are taken on a set of components of the index. It is common to see a short straddle or near-ATM strangle on the index and long similar straddles or strangles on 40% to 50% of the stocks that make up the index. If maximum dispersion is realized, the strategy will make money on both the long options on the individual stocks and on the short option position on the index, since the latter would have moved very little, earning theta. The strategy is evidently a low-premium strategy, with very low initial Delta with typically positive Vega bias.

The success of the strategy lies in determining which component stocks to pick. At the simplest level they should account for a large part of the index to keep the net risk low, but at the same time it is critical to make sure you are buying "cheap" volatility as well as candidates that are likely to "disperse."

Let's discuss the types of values that can be employed in the dispersion strategy. These values enable traders to determine whether current conditions are suitable for a dispersion trade. We will distinguish amongst these three kinds of values: realized, implied, and theoretical.

Realized values can be calculated on the basis of historical market data, e.g. prices observed on the market in the past. For example, values of historical volatility, correlations between stock prices are realized values.

Implied values are values implied by the option prices observed on the current day in the market. For example, implied volatility of stock or index is volatility implied by stock/index option prices, implied index correlation is an internal correlation implied by the market. We shall discuss this in further detail later on in the chapter.

Theoretical values are values calculated on the basis of some theory, so they depend on the theory you choose to calculate them. Index volatilities calculated on the basis of the portfolio risk formula are theoretical, and can differ from realized or implied volatilities.

A comparison of theoretical values with realized ones allows traders to determine what market behaviour are best applied to the actual trading environment. By studying and analyzing the historical relationship between these two types of values one can make an informed decision about related forecasts.

By comparing implied and historical volatilities, theoretical and realized, or theoretical and implied values of the index risk, one can attempt to ascertain the best time to employ the dispersion strategy or to choose to continue to monitor the markets.

The dispersion strategy typically consists of short selling options on a stock index while simultaneously buying options on the component stocks, the reverse dispersion strategy consist of buying options on a stock index and selling options on a the component stocks.
This is an example of a behavioural/stochastic arbitrage relationship and is the basis of the dissertation. In other words historical records show an *'a posteriori'* link between two derivatives. The relationship is based is principally based on correlation between different securities/derivatives or some form of dependence, but this association cannot be expected to resist all kinds of eventualities.

Later on we will discuss the behavioural/stochastic relationship between the two portfolios (i.e. between index options and the *n* individual options that are contained in the index). This arbitrage is called a non-linear index arbitrage, because options have a non-linear payoff structure.

Figure 1 illustrates the levels of implied and historic volatility on the Eurostoxx 50 Index. It can clearly be seen historic volatility was greater than implied volatility at times and reached a local maximum in the last week of September 2001. The explanation for this lies in the tragic events of September 11. A similar spike in volatility, which was driven by very different factors, can be observed in September 2002. This particular time was quite good for buying cheap index options, and thus to engage in the reverse dispersion strategy. Figure 2 displays the spread between the two measures over the same time horizon.

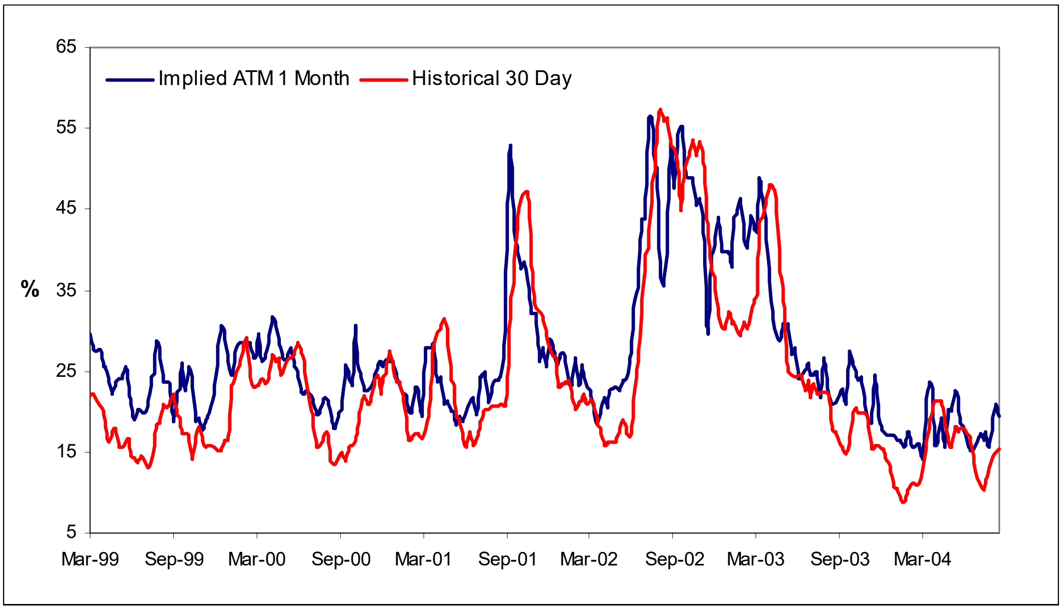

Figure 1. Levels of 1 month ATM Implied and 30 day historic volatilities on the Eurostoxx 50 Index

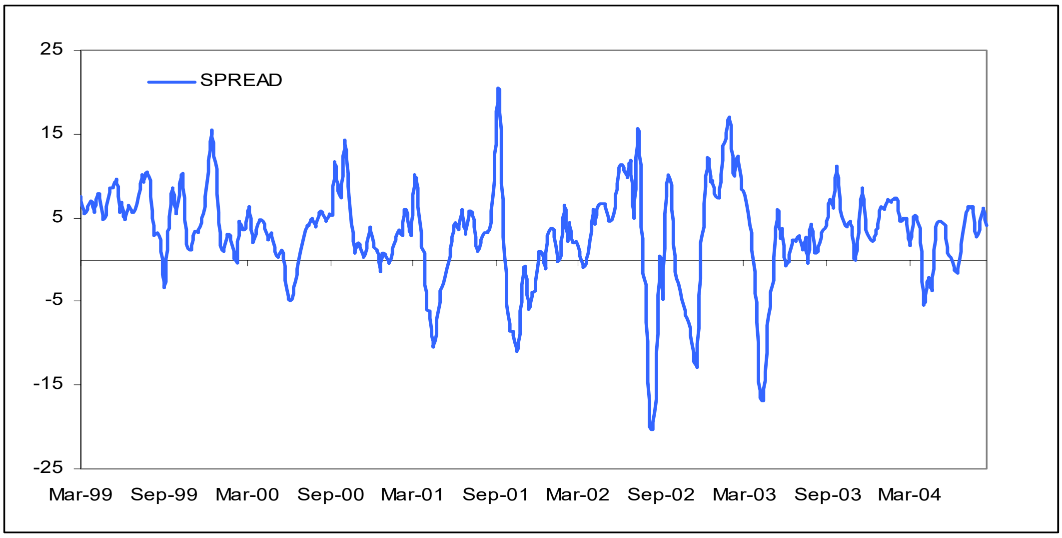

Figure 2. Volatility spread between Levels of 1 month ATM Implied and 30 day historic volatilities.

Characteristically, rising equity prices usually have the effect of cooling off the realised historical volatilities, and the implied volatilities tend to follow or anticipate that move, especially on the short end of the curve. Conventional wisdom has it that bear market moves tend to more volatile, with large spiky movements, whereas bull markets tend to be characterised by a steady appreciation in asset prices.

Empirical evidence has shown that in general implied volatilities on indices typically move synchronously. The implied volatility index of global equity indices such as the S&P 500, Dax, CAC40, FTSE100 and Nikkei are virtually coincident, since such indices reflect the state of economics as a whole, and so their performances are affected by analogous factors.
Another noteworthy point to make is that implied volatilities of global equity indices are lower than those of sector indices. This can be explained by the fact that changes in one stock within a sector can potentially have a considerable impact on the index of its sector, but the change may only have a slight influence on the overall market index. So, risk for an index that consists of equities from within the same industry is higher.

Major market indices have on the whole lower implied volatility in comparison with sector indices. But it should not be considered that some indices are better or worse for application of dispersion strategies. The important role in the strategy is not the absolute value of implied volatility, but the historical relationship between implied and realised index volatility.
The Eurostoxx 50 index is considered to be the most liquid of all European indices and hence the focus of our analysis will be primarily based on this.

1.2 Basket Options

Investors are typically interested in arbitrage possibilities between options on an index and options on the individual stocks. We will briefly discuss the pricing methodology that market practitioners use to price these options. Determining the price of a basket option is not a trivial task, because there is no explicit analytical expression available for the distribution of the weighted sum of the assets in the basket.

An option on such a basket is cheaper than buying options on each of the individual assets in the basket. The reason is that the volatility of the basket is less than the sum of the individual asset volatilities, unless the components' prices are perfectly correlated. Mathematically, this can be shown with a simple example: a basket that consists of two securities with volatilities σ_1 and σ_2, weights ω_1, ω_2 and correlated with coefficient ρ. Assuming that each of the assets exists in a Black Scholes economy and that their prices are distributed lognormally, the volatility of the basket, σ_B, is given by,

$$\sigma_B = \sqrt{\omega_1^2\sigma_1^2 + \omega_2^2\sigma_1^2 + 2\rho\omega_1\omega_2\sigma_1\sigma_2} \qquad (1)$$

When the two components are perfectly correlated, $\rho = 1$, and stocks weighted equally we obtain,

$$\sigma_B = \frac{1}{2}\sqrt{\sigma_1^2 + \sigma_1^2 + 2\sigma_1\sigma_2} = \frac{1}{2}\sqrt{(\sigma_1 + \sigma_2)^2} = \frac{\sigma_1 + \sigma_2}{2} \qquad (2)$$

which is the maximal possible value. (The correlation coefficient lies between minus one and one, by definition). From the financial perspective, the foregoing simple mathematical relations demonstrate the idea of diversification. The investor holds the basket to reduce the risk exposure compared with exposure to the individual assets. Hedging such a position with a set of options on the individual basket components works against the purpose. It overhedges the risk and costs too much.

However, to use an option on the basket as a whole, a pricing and hedging methodology is needed. The first step in using such a methodology consists of adopting a framework of underlying assumptions, and the most reasonable approach would be a straightforward extension of the familiar Black-Scholes framework onto the case of multiple assets.

What are possible approaches to pricing basket options in this setting? The simplest solution would be to treat the basket as a single asset—an index— and use familiar methods such as the Black-Scholes formula for European calls and puts to price the option.

The basket price at any given time t is a weighted sum of the prices $S_i(t)$ of *n* components,

$$B = \sum_{i=1}^{n} \omega_i S_i(t) \qquad (3)$$

where ω_i are the weights of each asset. We will assume that the world satisfies the conventional Black-Scholes assumptions, and each of the asset prices $S_i(t)$ follows a lognormal stochastic process,

$$dLnS_i(t) = S_i \exp\left[\left(\mu_i - \frac{\sigma_i^2}{2}\right)dt + \sigma_i dW_i(t)\right] \qquad (4)$$

where $dW_i(t)$ is a Wiener process. The solution of (1.1) is,

$$S_i(t) = S_i \exp\left[\left(\mu_i - \frac{\sigma_i^2}{2}\right)t + \sigma_i z_i \sqrt{t}\right] \qquad (5)$$

where $S_i = S_i(0)$ is the ith asset price at trade

σ_i is the ith annualised volatility

q_i is the corresponding continuously compounded, annualised dividend yield

$\mu_i = r - q_i$ is the mean rate of return on the ith asset in a risk-neutral world

r is the continuously compounded, annualised riskless interest rate and

x_i is a random variable, normally distributed with mean 0 and variance 1.

We shall also assume the logarithms of assets i and j are correlated with coefficient ρ_{ij} i.e. $E\lfloor z_i z_j \rfloor = \rho_{ij}$. Note that the mean α_i and the standard deviation β_i of $\ln S_i(t)$ are given by,

$$\alpha_i(t) = \ln S_i + \mu_i t_i \qquad \beta_i(t) = \sigma_i \sqrt{t} \qquad (6)$$

In this framework, the value of a European option on a basket is given by

$$V = e^{-rt} E\left\{\max\left(\phi\left[\sum_{i=1}^{n} \omega_i S_i(T) - K\right], 0\right)\right\} \qquad (7)$$

where T is the option expiry time, K is the strike price, $E[\bullet]$ denotes the risk-neutral expectation value, and ϕ is 1 for a call and –1 for a put.

While simplicity is an important advantage, this method has obvious drawbacks. If the basket components' prices are lognormally distributed, then assuming that the distribution of basket prices is also lognormal is inconsistent. An even bigger drawback of this approach is that it makes it impossible to hedge exposure to the individual volatilities and correlations.

Investors seek arbitrage possibilities between options on an index and options on individual stocks within the index. The trade based on this strategy is intended to exploit the discrepancy between the pricing of index options and the portfolio of individual stock options. This discrepancy can be found in the difference in correlation or volatility in almost equal