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2014 Mathematical Contest in Modeling (MCM) Summary Sheet

Summary

The keep-right-except-to-pass rule is widely implemented all around the world, but it may not be an optimum one. We define five evaluation criteria to evaluate the performance of a traffic rule, namely, the average traffic speed, traffic flow, danger index, the over-speed-limit effect and the under-speed-limit effect. To analyze the "keep right" rule's performance theoretically, we apply a state transition approach, which is similar to the Markov model, in light traffic situation. The result shows that all the cars will travel in the right lane at a low speed in the long term. In heavy traffic, we analyze its steady state and discover that cars will break the "keep right" rule because they cannot find a chance to return to the right lane after overtaking. To test the theoretical results, we build a simulation model based on the Cellular Automation (CA) to simulate the traffic system under a given traffic rule. The simulation results are consistent with what we have got through the state transition approach.

In order to seek a better traffic rule, we develop 3 new rules based on the old rule. Then we evaluate their performance together with the old rule respectively using our CA-based simulator. After calculating the values of our evaluation criteria, we employ the Analytic Hierarchy Process (AHP) method to obtain the best solution. We find that the best rule is the one which forbids cars from overtaking to achieve the best safety performance and highest traffic flow.

Finally, we discuss some further topics. The result is that we can apply our best solution to left-hand traffic countries by simply changing the orientation, and that the application of intelligent system will improve the performance of the "keep right" system in light traffic, but deteriorate it in heavy traffic. Results of the sensitivity analysis based on the CA simulator have shown the robustness of our conclusions.

Our suggestion for the public is that everyone should consciously avoid the overtaking behavior to realize a better traffic condition. Further studies should focus on more complex circumstances such as the six-lane freeway. With more precise data available, we can further test and improve our models.

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1 Introduction

Recent changes in economics and technology have enabled more and more households to own their private cars, but at the same time, have posed more pressure on highway capacity. It is necessary for the government to implement proper traffic rules which can maximize the traffic flow while ensuring safety. The most popular traffic rule for the time being is the keep-right-except-to-pass rule, which states that all drivers should drive in the right-most lane unless they are passing another vehicle, and when overtaking, drivers should move one lane to the left to pass the vehicle ahead and return to the right lane as soon as possible. This rule is widely implemented in most countries in the world, including the U.S and China. In Great Britain and some other countries, this rule is adjusted with a simple change of orientation, and we can call it the keep-left-except-to-pass rule.

However, is the "keep right" rule optimum? Or, is there any alternative traffic rule superior to this one in terms of traffic flow, safety, and other important factors? This is one main issue we want to deal with in this paper. In fact, we can divide the whole problem into four major subproblems:

1. **In both light and heavy traffic, what is the performance of the keep-right-except-to-pass rule?** This requires us to set up several evaluation criteria. Based on the answer to this question, we can decide whether to design a new traffic rule to replace, and in which aspects improvement can be made.

2. **Is there a better traffic rule? If yes, why can we say the new rule is better?** We must integrate our evaluation criteria into a comprehensive one to decide between rules. This involves the determination of weights and a comparison of the two traffic rules.

3. **Does the new rule apply to left-hand traffic countries?** Except for a simple change of orientation, we should decide whether some other requirements need to be met.

4. **Would there be any change in our analysis results if all the vehicles were under the control of an intelligent system?** In fact it is an optimization problem. We can control the behavior of each vehicle in the freeway, like determining whether it should change lanes. In this case, we may achieve the best traffic condition.

For the rest of our paper, we will first set up five criteria to evaluate traffic rules. Then we look into the keep-right-except-to-pass rule. We use a state transition approach to study its performance in light traffic, and the heavy traffic situation is also considered. Next, a simulation model based on the cellular automation is built to verify the results given by the theoretical model. Afterwards, we design three different new rules and evaluate them together with the old rule, where we use the AHP method to get the best solution. Finally, we discuss further topics about the rule under left-hand traffic and the old performance under control of the intelligent driving system. The basic logic framework of our paper is shown in **Figure 1**.

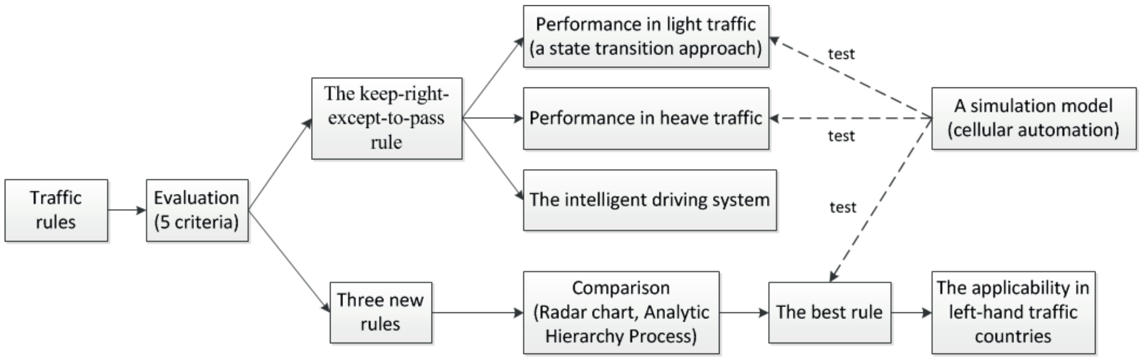

Figure 1: The logic framework of our paper.

2 General Assumptions

- **Drivers consciously drive their cars and everything is in normal state.** In other words, the drivers are rational and the road condition is good. We do not consider abnormal situations where drivers are drunk or sleepy while driving, or the road is frozen and slippery because of the extreme weather.

- **In our models, we only consider countries where driving automobiles on the right is the norm.** Driving on the right is different from driving in the right lane, since drivers can also drive in the left lane on the right in multi-lane traffic. The research object of this paper is the rule which requires drivers to drive in the right lane except to overtake. For completeness, we will consider the rules in countries where driving on the left is the norm in subsection **6.1**.

- **For simplicity, we only consider a four-lane freeway(two in each direction), with a center dividing strip between the opposing traffic flows.** The freeway has only two lanes for each direction, the right one for driving and the left one for overtaking only(under the keep-right-except-to-pass rule). What's more, the center dividing strip ensures that there is no interactions between vehicles traveling in opposite directions.

- **There are no stop lights or intersections to interrupt the flow of traffic. Also, there are no other entrances or exits, and no sharp turns.** Vehicles come from only one entrance and drive their way through. Because vehicles behave differently in sharp turns, we overlook such area. This simplicity makes little difference to our results.

- **There is only one type of vehicle in the freeway.** All vehicles have the same brand, model and size(especially length). In other words, they are homogeneous. For convenience, we call them "cars".

- **When traveling individually, cars move at a constant speed(free speed). But when encountering another car ahead, cars can change speed instantly.** This assumption means that we overlook the accelerating process of cars. They can change speed with no time. But when traveling individually, they will all move at their own free speed.

- **Cars enter the freeway in a Poisson manner.** That is to say, when two neighboring cars arrive at the beginning point of our freeway sequently, the time interval is exponentially distributed.

3 The Keep-Right-Except-To-Pass Rule

Many methods can be used to describe vehicular traffic flow at low and moderate densities on long, uninterrupted freeways, including queuing theory, Markov chains, cellular automation, traffic flow differential equations and so on. The first two are macroscopic and the last one is microscopic, while cellular automation is often used to do the simulation[2][12].

In this section, our main task is to analyze the performance of the keep-right-except-to-pass rule. We first choose five statistics as our evaluation criteria to assess traffic rules. Then we use a state transition approach(similar to the Markov model) to analyze this rule in light traffic. Next, we extend the model to point out some important problems in heavy traffic. Afterwards, we employ the cellular automation method to simulate the behavior of cars in freeway. We calculate the values of our five evaluating criteria and give some interpretations. Finally, we analyze the sensitivity of our simulation model. This section is the basis of the entire paper, our further analysis, comparison and adjustment are to some extent dependent on the methods employed here.

Before start, we shall mention the definition of “light” and “heavy” traffic here. In common sense, when the number of vehicles in the freeway is very huge, we call the situation as “heavy traffic”. But what does “huge” mean? This definition is rather vague and not appropriate for research use. In this paper, we state “light” and “heavy” in this way: **if a car which changes lanes to overtake cannot find a place to return to its initial lane within certain time range, the traffic is heavy. Otherwise the traffic is light.** In section **3**, we will use the variable λ (arrival rate) to illustrate. The closer λ is to 0.5, the heavier the traffic is.

3.1 Traffic Rule Evaluation

We must set up several basic criteria to analyze the performance of the keep-right-except-to-pass rule. These evaluation criteria include both static ones(such as safety) and comparative static ones(such as performance in extreme conditions). Also, the criteria should be easily calculated from available data. We choose the following five evaluation criteria:

- **Traffic flow**: The number of cars passing an observing point per unit of time. Here we set its unit as “vehicles per second”.

- **Danger index**: The average number of lane changes for each car in our assumed freeway. **It measures safety**. Overtaking is risky because the car behind may crash into the car ahead when changing lanes. The more frequently a car changes lanes, the more likely an accident may take place. In contrast, when a car travels in a fixed lane, it can adjust speed instantly when encountering another car ahead, so it is not possible that an crashing accident took place. For simplicity, we assume that **accidents will happen only when the car changes lanes**. So we can use the number of lane changes per car as a measure of safety. Its unit is “times per vehicle”.

- **Average traffic speed**: The average speed of all cars passing an observing point. Its unit is “kilometer per hour”. It measures how fast cars are under a specific traffic rule.

- **USL effect**: It stands for “Under-posted Speed Limit effect”. Celeris paribus, if the traffic flow in a situation where the speed limit is too low is a, and the traffic flow in a situation where the speed limit is moderate is b, then the USL effect equals the ratio of a to b. It measures the performance of a traffic rule in extreme conditions. If the traffic flow decreases too sharply when speed limit is too low or too high, we do not regard the traffic rule as a good one.

- **OSL effect**: Accordingly, it stands for “Over-posted Speed Limit effect”. Its definition and function are similar to those of the USL effect.

Then how to evaluate a specific traffic rule? Since the traffic flow and the average traffic speed both measure the efficiency and capacity of freeways, the larger they are, the better the rule performs. For the danger index, we hope it to be as small as possible so as to decrease the probability of an accident. USL and OSL effects measure the rule’s role in extreme conditions. What we want is that the traffic flow of the freeway was still relatively high when the speed limit was too low or too high. So these two effects of a good traffic rule should also be large.

3.2 Performance In Light Traffic

In light traffic condition, there are not many cars, so the distance between neighboring cars is big. Consequently, it is comparatively easy to change lanes, overtake and return without worrying about being collided or finding nowhere to return. In fact, the traffic state at time t can be computed from the traffic state at time 1 through multiple iterations. This is similar to the Markov model, but the difference lies in that the transition probability matrix is not constant. Following this thinking, we use a state transition approach to look for a steady state of traffic flow.

In addition to the general assumptions mentioned above, we consider here the following situation: all the cars enter the system through the right lane. According to speed, they are divided into 3 discrete states: State 1, State 2 and State 3. The speed of a car in State i is different from that in State j if $i \neq j$. After overtaking the car ahead , the car at a higher speed will return to the right-most lane as soon as possible, and travel at its previous speed v_i, because the drivers may prefer to different speeds in the freeway. If a car chooses not to overtake, it will change to the same state with the preceding car, avoiding collision. For simplicity, we arbitrarily ignore the accelerating process of overtaking. In other words, the overtaking process is finished immediately.

3.2.1 Notations

- v_i: The speed of cars in State i, $i = 1, 2, 3$. For $i > j$, we have $v_i < v_j$.
- v_4: The speed of cars in the left lane. Whatever the car's free speed is, if it changes lanes to the left, it has to travel at the speed of v_4. We assume $v_4 > max(v_1, v_2, v_3)$ for the convenience of overtaking.
- $\pi_k^{(t)}$: The proportion of cars in state k after t times of transitions.
- p_{ij}: The probability that a car in State i moves to State j. According to subsection **3.3**, $j \geq i$.

3.2.2 A State Transition Approach

According to FRESIM(the freeway model within the CORSIM software), the probability of overtaking when a car catches up with another in the right lane positively correlate with their relative speed. For convenience, let p_1 represent the overtaking probability of when State 1 meets State 2, that is, $p_1 = p(v_1 - v_2)$, where $p > 0$ is a proportional coefficient. Similarly, $p_2 = p(v_1 - v_3)$, $p_3 = p(v_2 - v_3)$.

Figure 2 illustrates how an overtaking takes place. The car behind follows a probability distribution to decide whether to follow or overtake. If it choose to follow, its speed must slow down to avoid collision.

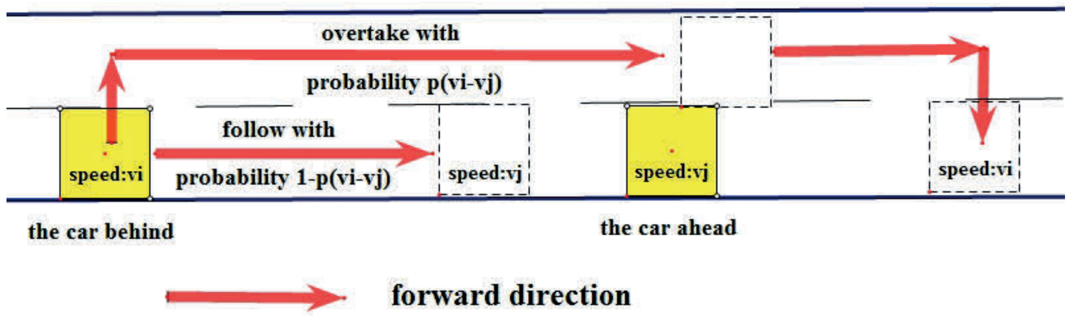

Figure 2: The choice between following and overtaking(i¡j).

Initially, the proportions of cars in State 1,2 and 3 are $\pi_1^{(1)}$, $\pi_2^{(1)}$, $\pi_3^{(1)}$, respectively. Using the Bayes theorem, we can get the probability of Sate 1 maintaining its state:

$$p_{11} = \frac{\pi_2^{(1)}}{\pi_2^{(1)} + \pi_3^{(1)}} p_1 + \frac{\pi_3^{(1)}}{\pi_2^{(1)} + \pi_3^{(1)}} p_2$$

Similarly, we can get all p_{ij}. Putting them together, the transition probability matrix is

$$\boldsymbol{T^{(1)}} = \begin{pmatrix} \frac{\pi_2^{(1)}}{\pi_2^{(1)}+\pi_3^{(1)}}p_1 + \frac{\pi_3^{(1)}}{\pi_2^{(1)}+\pi_3^{(1)}}p_2 & \frac{\pi_2^{(1)}}{\pi_2^{(1)}+\pi_3^{(1)}}(1-p_1) & \frac{\pi_3^{(1)}}{\pi_2^{(1)}+\pi_3^{(1)}}(1-p_2) \\ 0 & p_3 & 1-p_3 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

Given the initial proportion vector $\boldsymbol{\pi^{(1)}} = (\pi_1^{(1)}, \pi_2^{(1)}, \pi_3^{(1)})$ and the transition probability matrix $\boldsymbol{T^{(1)}}$, we can infer that if a car catches up with another in the system, the proportion vector will be updated like the following:

$$\boldsymbol{\pi^{(2)}} = (\pi_1^{(2)}, \pi_2^{(2)}, \pi_3^{(2)}) = \boldsymbol{\pi^{(1)}} \cdot \boldsymbol{T^{(1)}} \tag{2}$$

Plugging (1) into (2), then extracting the first component of the vector, yields

$$\pi_1^{(2)} = \left[\frac{\pi_2^{(1)}}{\pi_2^{(1)}+\pi_3^{(1)}}p_1 + \frac{\pi_3^{(1)}}{\pi_2^{(1)}+\pi_3^{(1)}}p_2\right] \cdot \pi_1^{(1)}$$

The transition probability matrix will also be updated because the proportion of cars in different states will change after one-time transition. The iteration formula is $\boldsymbol{\pi^{(n+1)}} = \boldsymbol{\pi^{(n)}} \cdot \boldsymbol{T^{(n)}}$. Repeating this process, we get

$$\pi_1^{(n+1)} = \left[\frac{\pi_2^{(n)}}{\pi_2^{(n)}+\pi_3^{(n)}}p_1 + \frac{\pi_3^{(n)}}{\pi_2^{(n)}+\pi_3^{(n)}}p_2\right] \cdot \pi_1^{(n)} \tag{3}$$

Due to the property of simple average, we have

$$\frac{\pi_2^{(n)}}{\pi_2^{(n)}+\pi_3^{(n)}}p_1 + \frac{\pi_3^{(n)}}{\pi_2^{(n)}+\pi_3^{(n)}}p_2 \leq max(p_1, p_2) < 1 \tag{4}$$

So, plugging (4) into (3) and iterating for n times, we can get

$$0 \leq \pi_1^{(n+1)} \leq [max(p_1, p_2)]^n \cdot \pi_1^{(1)}$$

Taking the limit, then using the squeeze theorem, yields

$$\lim_{n\to\infty} \pi_1^{(n+1)} = 0$$

That means, after N times of transitions, the proportion of State 1 cars is zero if N approaches to infinity. Thus, $\boldsymbol{\pi^{(n)}} = (0, \pi_2^{(n)}, \pi_3^{(n)})$, and the transition probability matrix between State 2 and State 3 is

$$\boldsymbol{T_{23}^N} = \begin{pmatrix} p_3 & 1-p_3 \\ 0 & 1 \end{pmatrix}$$

From $\boldsymbol{T_{23}^N}$, we can see that State 2 will transfer to State 3 with the probability of $1 - p_3$, while State 3 can only maintain State 3 itself. Thus State 3 is an absorbing state. In other words, after $N + M$ times of transitions, all cars will be in State 3(the lowest speed) if M approaches to infinity, that is

$$\lim_{M,N\to\infty} \boldsymbol{\pi}^{(\boldsymbol{N+M})} = (0, 0, 1)$$

3.2.3 A Numerical Test And Conclusion

We use a numerical method to simulate the iterating and matrix updating processes mentioned above, so as to test our model results.

We might as well set p_1=0.35, p_2=0.45, p_3=0.29.From our notations, $\pi_k^{(t)}$ is the proportion of cars in state k after t times of transitions, and we set their initial values as following: $\pi_1^{(1)}$=0.2, $\pi_2^{(1)}$=0.5, $\pi_3^{(1)}$=0.3. Then we use Matlab to get the following simulation figure:

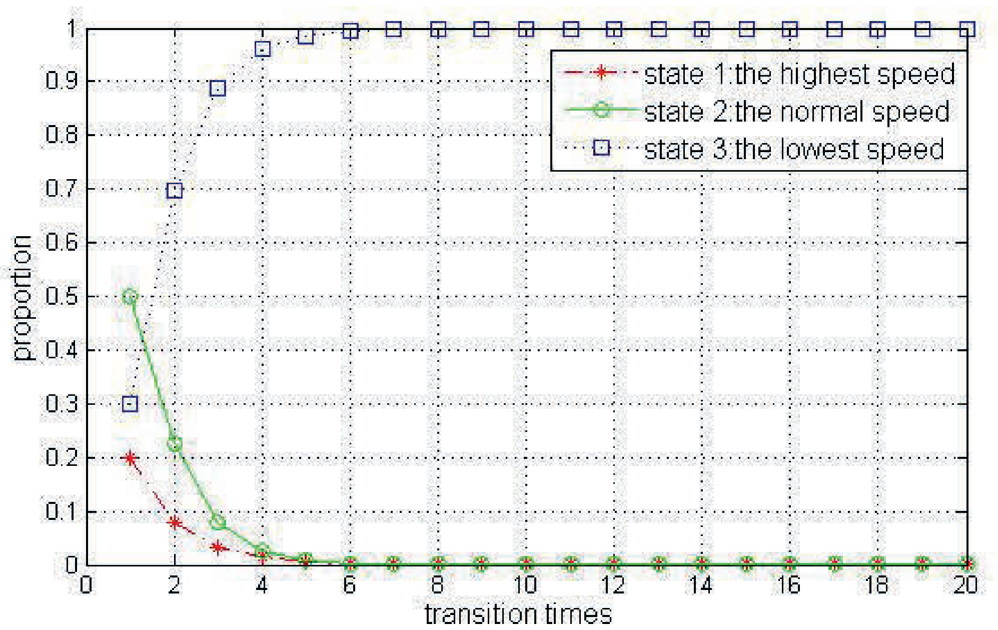

Figure 3: Proportions of cars in different states after several transitions.

From **Figure 3** we can clearly see that the proportion vector equals to (0, 0, 1) after 6 times of transitions. This simulation result is consistent with our model. Later, we will employ a simulation method to test this result again in subsection **3.5**.

According to our above analysis, we can confidently conclude that if all the cars in the freeway obey the keep-right-except-to-pass rule, they will all travel in the right lane at a relatively low speed in the long term, while at the same time, the left lane is comparatively empty. **So, the traffic flow, one of our evaluation criteria, under the "keep right" rule is small.** In order to make better use of