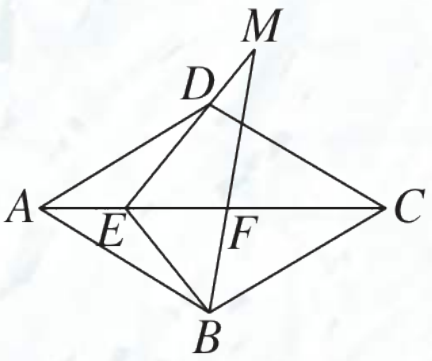
阶段拔尖专训4 在特殊四边形中的结论判断

题型1 在菱形中的结论判断

1.[2024·菏泽期中] 如图，点E，F在菱形$ABCD$的对角线AC上，$\angle ADC=120^\circ$，$\angle BEC=\angle CBF=50^\circ$，$ED$与$BF$的延长线交于点$M$．以下结论：

①$\angle BME=30^\circ$；

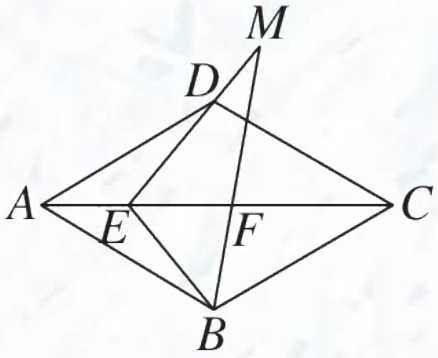
②$\triangle ADE\cong\triangle ABE$;



（第1题）

③$EM = BC$.

其中正确结论的个数是(D)



（第1题）

A.0　　B.1　　C.2　　D.3

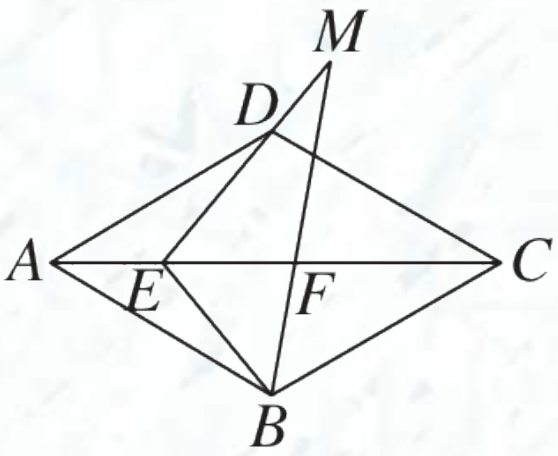
【点拨】∵ 四边形$ABCD$是菱形，

$\angle ADC = 120^\circ$ ，

$\therefore AD = AB = BC = CD$，

$\angle BCD = 60^\circ$ ，$\angle DAE = \angle BAE = \angle DCE = \angle BCE$. $\therefore \angle BCE = 30^\circ$.

$\therefore \angle BFE = \angle BCE + \angle CBF = 30^\circ + 50^\circ = 80^\circ$.



（第1题）

$\therefore \angle EBF = 180^\circ - \angle BEC - \angle BFE =$

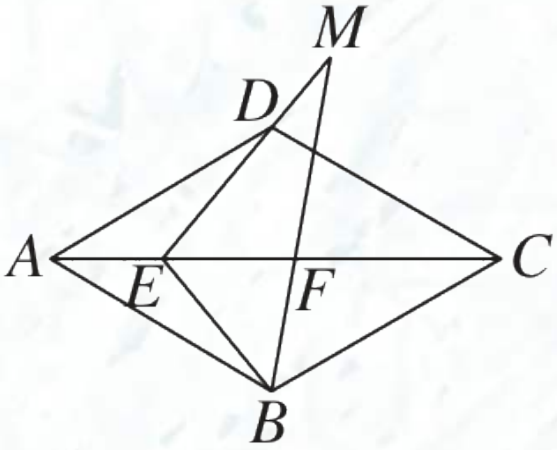
$180^\circ - 50^\circ - 80^\circ = 50^\circ$.

在$\triangle CDE$和$\triangle CBE$中，$\begin{cases} CD = CB, \\ \angle DCE = \angle BCE, \\ CE = CE, \end{cases}$

$\therefore \triangle CDE \cong \triangle CBE(\mathrm{SAS})$. $\therefore \angle DEC =$

$\angle BEC = 50^\circ$.

$\therefore \angle BEM = \angle DEC + \angle BEC = 100^\circ$.



（第1题）

$\therefore \angle BME = 180^\circ - \angle BEM - \angle EBF =$

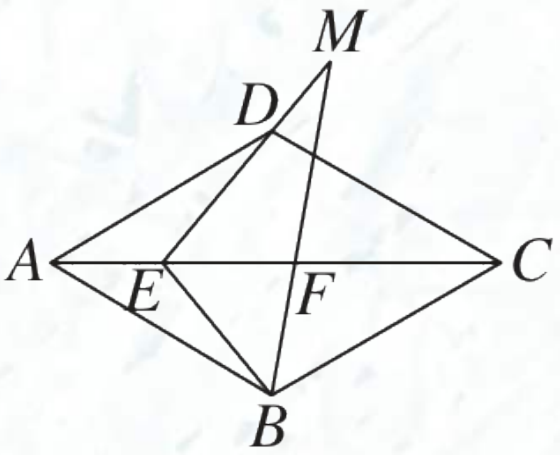
$180^\circ - 100^\circ - 50^\circ = 30^\circ$.故①正确;

在$\triangle ADE$和$\triangle ABE$中,

$$\begin{cases} AD = AB, \\ \angle DAE = \angle BAE, \\ AE = AE, \end{cases}$$

$\therefore \triangle ADE \cong \triangle ABE(\mathrm{SAS})$.故②正确;

$\because \angle EBC = \angle EBF + \angle CBF = 100^\circ$,



（第1题）

$\therefore \angle BEM = \angle EBC.$

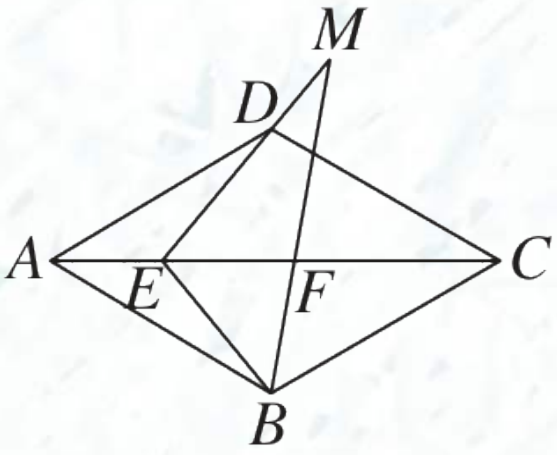
在$\triangle BEM$和$\triangle EBC$中，

$$\begin{cases} \angle BEM = \angle EBC, \\ \angle BME = \angle ECB = 30^\circ, \\ BE = EB, \end{cases}$$

$\therefore \triangle BEM \cong \triangle EBC(\text{AAS}).$

$\therefore EM = BC.$故③正确.

$\therefore$ 正确结论的个数是3，故选D.



（第1题）

2.[2024·青岛阶段练习] 如图，在菱形$ABCD$中，$\angle A=60^{\circ}$，E，F分别是AB，AD的中点，DE，BF相交于点G，连接BD，CG．有下列结论：

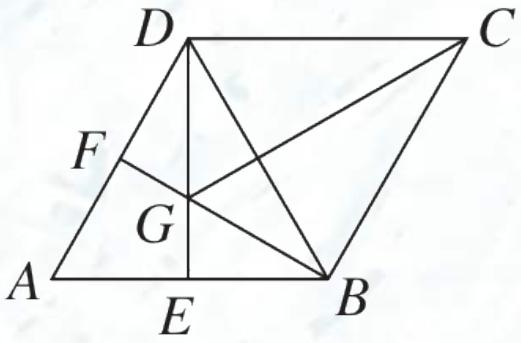
①$\angle BGD=120^{\circ}$；

②$BG+DG=CG$；

③$\triangle BDF\cong\triangle CGB$.

其中正确的结论有(C)

A.0个　　B.1个　　C.2个　　D.3个



（第2题）

【点拨】∵ 四边形$ABCD$是菱形，

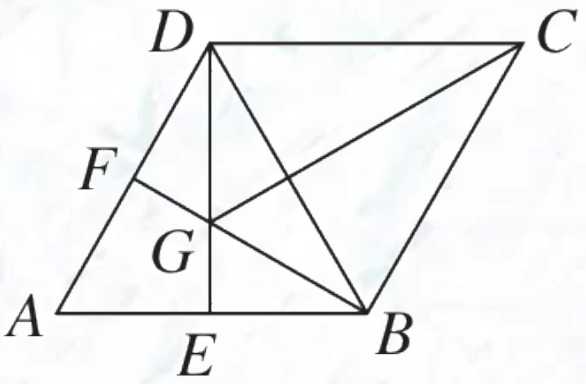
$\angle A = 60^\circ$，

$\therefore AB//CD, AD//BC$,

$AB = BC = CD = AD, \angle BCD = 60^\circ$.

$\therefore \triangle ABD$是等边三角形，$\triangle BDC$是等边三角形.

$\therefore \angle ADB = \angle ABD = 60^\circ$，$BC = BD$.



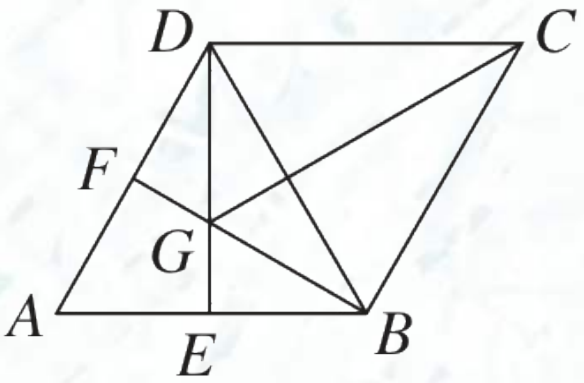
（第2题）

$\because E$，F分别是AB，AD的中点，

$\therefore \angle BFD = \angle DEB = 90^\circ$.

$\therefore \angle GDB = \angle GBD = 30^\circ$.

$\therefore GB = GD, \angle BGD = 180^\circ - 30^\circ - 30^\circ = 120^\circ$.故①正确；



（第2题）

在$\triangle CDG$和$\triangle CBG$中，$\begin{cases} CD = CB, \\ CG = CG, \\ DG = BG, \end{cases}$

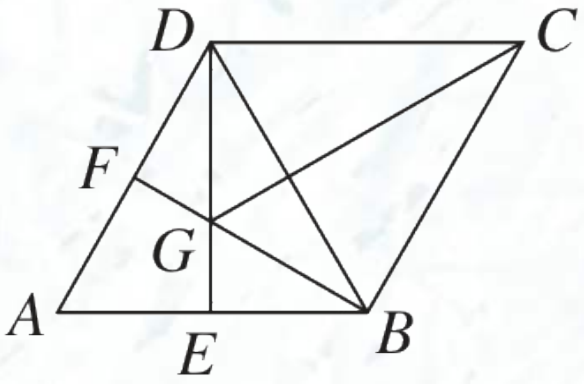
$\therefore \triangle CDG \cong \triangle CBG(\mathrm{SSS})$. $\therefore \angle DGC = \angle BGC = 60^\circ$.

$\because \angle BFD = \angle DEB = 90^\circ$, $AB//CD$, $AD//BC$,

$\therefore \angle GDC = \angle GBC = 90^\circ$.

$\therefore \angle GCD = 30^\circ$.

$\therefore CG = 2GD = DG + BG$. 故②正确.

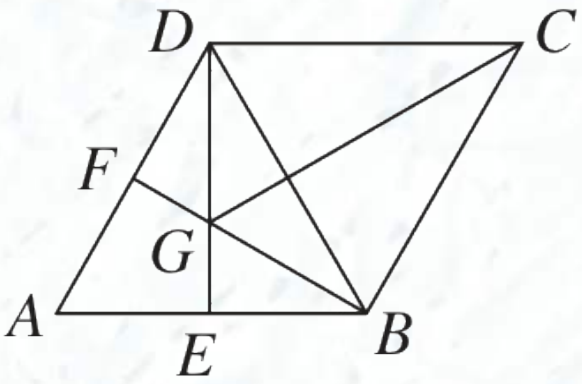


（第2题）

$\because \triangle GBC$为直角三角形，$\therefore CG > BC$.

$\therefore CG \neq BD$. $\therefore \triangle BDF$与$\triangle CGB$不全等．故③错误；

$\therefore$ 正确的结论有①②，共2个．故选C．

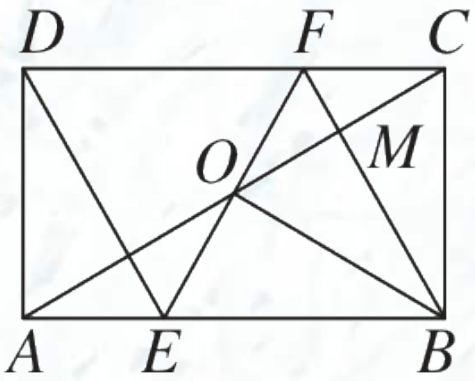


（第2题）

题型2 在矩形中的结论判断

3.如图，矩形$ABCD$中，O为AC的中点，过点O的直线分别与AB，CD交于点E，F，连接BF交AC于点M，连接DE，BO.若$\angle COB=60^{\circ}$，$FO=FC$，则下列结论：

①$FB\perp OC$；



（第3题）

②$\triangle EOB \cong \triangle CMB$；

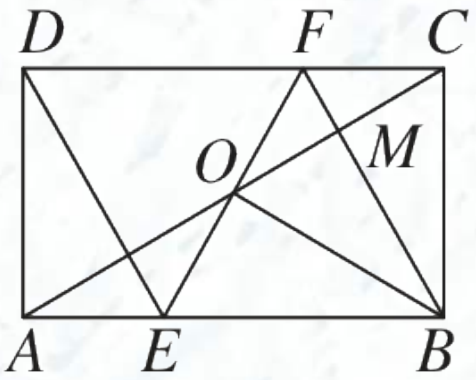
③四边形$EBFD$是菱形；

④$MB:OE=3:2$.

其中正确结论的个数是(　B　)

A.4　　B.3　　C.2

D.1



（第3题）

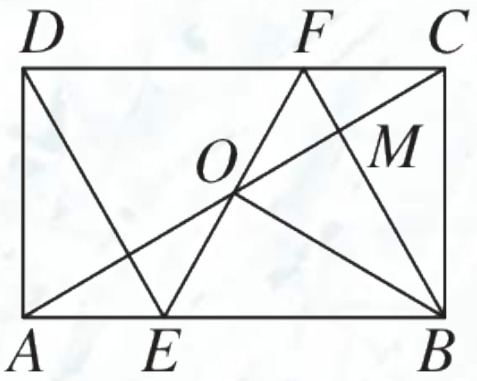
【点拨】∵ 四边形$ABCD$是矩形，O为AC的中点，

$\therefore AB = CD, AB // CD,$

$\angle ABC = \angle BCD = 90^\circ$,$OA = OB = OC.$

又∵ $\angle COB = 60^\circ$ ，∴ $\triangle OBC$是等边三角形.

$\therefore \angle ACB = \angle OBC = 60^\circ$ ，$BC = OB.$



（第3题）

又$\because FO = FC$，$\therefore FB \perp OC$，故①正确；

$\therefore \angle CBM = \angle MBO = 30^\circ$.

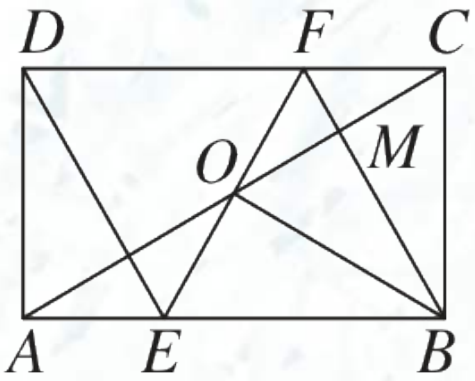
$\because \angle ABC = \angle BCD = 90^\circ$,

$\angle OBC = \angle OCB = 60^\circ$,

$\therefore \angle ABO = 30^\circ$,$\angle ACD = 30^\circ$.

$\therefore \angle ABF = 60^\circ$,

$\angle EFB = \angle CFM = 90^\circ - 30^\circ = 60^\circ$.



（第3题）

$\therefore \triangle EFB$是等边三角形.$\therefore BE = BF$.

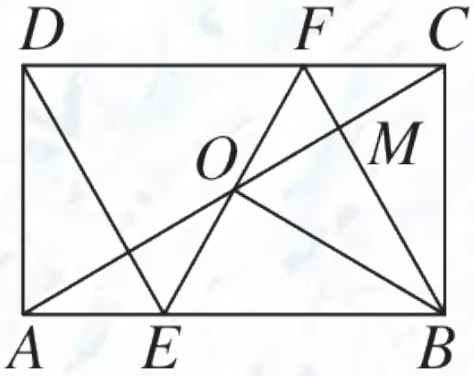
$\because AB//CD, \therefore \angle ACD = \angle CAB$.

在$\triangle FOC$和$\triangle EOA$中，$\begin{cases} \angle FOC = \angle EOA, \\ \angle FCO = \angle EAO, \\ OC = OA, \end{cases}$

$\therefore \triangle FOC \cong \triangle EOA$(AAS). $\therefore AE = CF$,

$OE = OF$.

$\because DC = AB$，$\therefore DF = EB$.



（第3题）

又$\because DF//EB$，$\therefore$ 四边形$EBFD$是平行四边形.

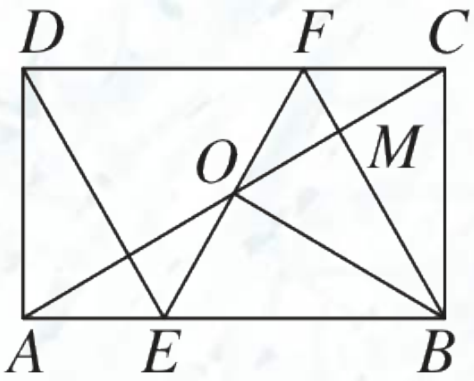
又$\because BE=BF$，$\therefore$ 四边形$EBFD$是菱形，

故③正确;

$\because BE=BF$，$BO=BC$，

$\angle EBO=\angle FBC=30^{\circ}$，

$\therefore \triangle EOB\cong\triangle FCB$.



（第3题）

$\therefore \triangle EOB$与$\triangle CMB$不全等，故②错误;

设$FM = a$.

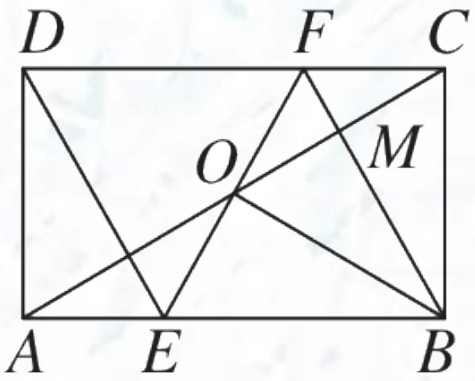
在Rt$\triangle OFM$中，$\because \angle OFM = 60^\circ$，

$\therefore \angle FOM = 30^\circ$.

$\therefore OE = OF = 2FM = 2a$.

$\because \angle FOM = 30^\circ$，$\angle COB = 60^\circ$，

$\therefore \angle FOB = 90^\circ$.



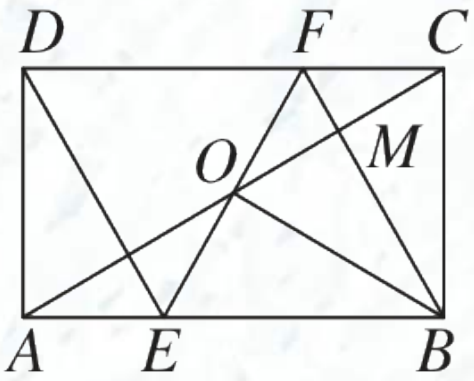
（第3题）

在Rt△FOB中，$\because \angle FBO = 30°$，

$\therefore BF = 2OF = 4a. \therefore BM = 3a.$

$\therefore BM : OE = 3 : 2$，故④正确.

综上，正确的结论有①③④，共3个，故选B.



（第3题）