

结构振动的最优控制

(最优控制设计--控制律设计：伺服机问题算例)

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$$2n \text{ 阶结构振动控制系统: } \begin{cases} \dot{\mathbf{y}} = \mathbf{A}\mathbf{y} + \mathbf{B}\mathbf{u}(t), & \mathbf{y}(t_0) = \mathbf{y}_0 \\ \mathbf{z} = \mathbf{E}\mathbf{y} \end{cases}$$

$\mathbf{y} \in \mathfrak{R}^{2n \times 1}$ 状态变量列阵 $\mathbf{A} \in \mathfrak{R}^{2n \times 2n}$ 系统系数矩阵 $\mathbf{B} \in \mathfrak{R}^{2n \times r}$ 控制位置指示矩阵

$\mathbf{u}(t) \in \mathfrak{R}^{r \times 1}$ 控制力列阵 $\mathbf{z} \in \mathfrak{R}^{l \times 1}$ 观测输出列阵 $\mathbf{E} \in \mathfrak{R}^{l \times 2n}$ 观测输出系数矩阵

$$\text{性能指标: } J = \frac{1}{2} \int_{t_0}^{\infty} \left[(\mathbf{z} - \mathbf{y}_d)^T \mathbf{Q} (\mathbf{z} - \mathbf{y}_d) + \mathbf{u}^T \mathbf{R} \mathbf{u} \right] dt \quad \mathbf{Q} \in \mathfrak{R}^{2n \times 2n} \quad \mathbf{R} \in \mathfrak{R}^{r \times r}$$

$$\text{最优跟踪控制律: } \mathbf{u}^*(t) = \underbrace{-\mathbf{R}^{-1} \mathbf{B}^T \mathbf{Y} \mathbf{y}(t)}_{\text{负反馈的调节作用}} + \underbrace{\mathbf{R}^{-1} \mathbf{B}^T [\mathbf{Y} \mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T - \mathbf{A}^T]^{-1} \mathbf{E}^T \mathbf{Q} \mathbf{y}_d}_{\text{由跟踪值 } \mathbf{y}_d \text{ 所导致的驱动作用}}$$

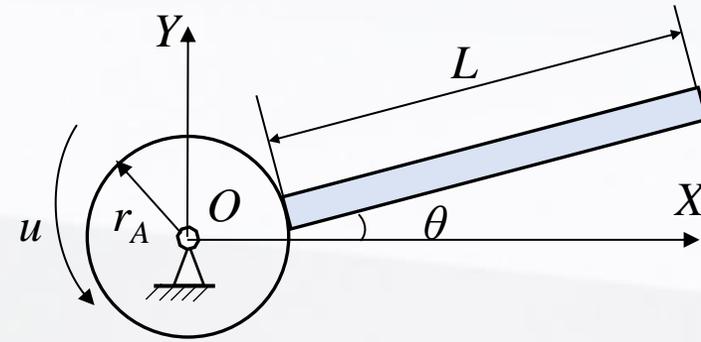
负反馈的调节作用 由跟踪值 \mathbf{y}_d 所导致的驱动作用

$$\text{李雅普诺夫方程: } \mathbf{Y} \mathbf{A} + \mathbf{A}^T \mathbf{Y} - \mathbf{Y} \mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T \mathbf{Y} + \mathbf{E}^T \mathbf{Q} \mathbf{E} = \mathbf{0}$$

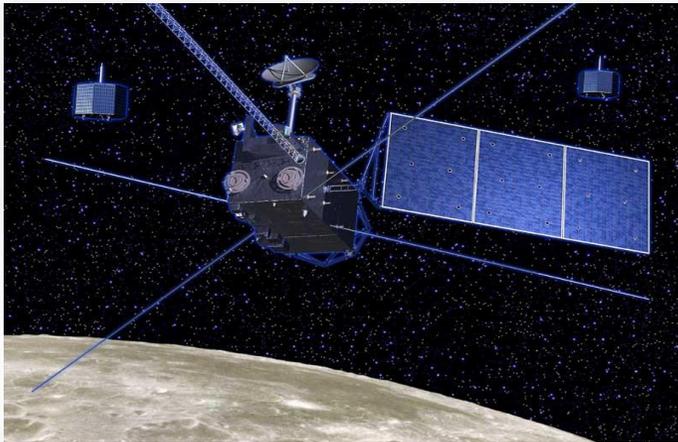
若状态全部已知, 则 \mathbf{E} 为单位阵

【例】 中心刚体-悬臂梁系统

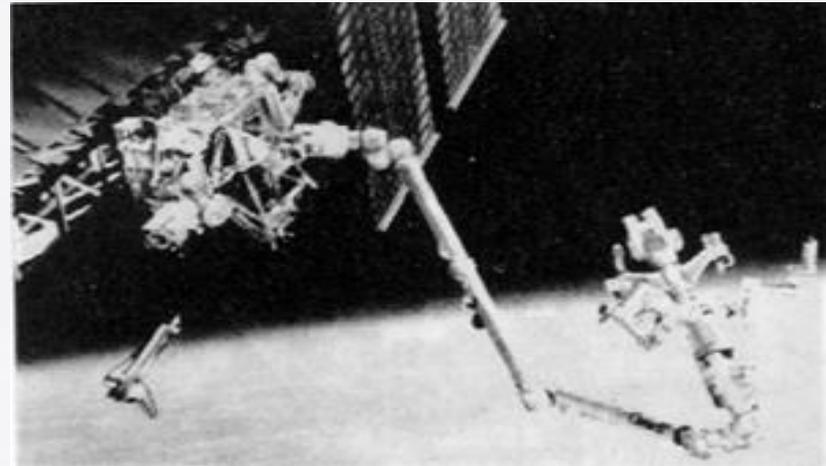
工程背景： 航天器， 机械臂，
大型涡轮机叶片等



航天器



柔性机械臂

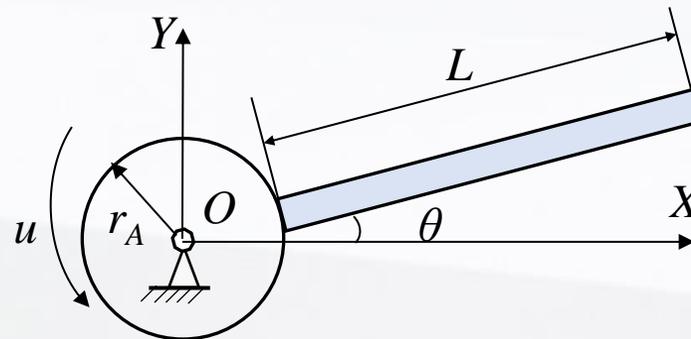


涡轮机叶片



【例】 中心刚体-悬臂梁系统

工程背景： 航天器，机械臂，
大型涡轮机叶片等



系统做平面大范围旋转运动的同时，也将产生柔性梁自身的弹性振动，这两种运动相互耦合、相互影响，构成了**刚柔耦合动力学系统**

梁长 L ，弹性模量 E ，截面惯性矩 I ，梁密度 ρ ，横截面积 A ，中心刚体半径 r_A

试设计最优控制器，以实现点-点运动控制和旋转运动控制

点-点运动控制： 控制系统由某一位置到达另一位置

旋转运动控制： 控制系统按某一指定角速度进行旋转运动

中心刚体-柔性梁系统动力学建模

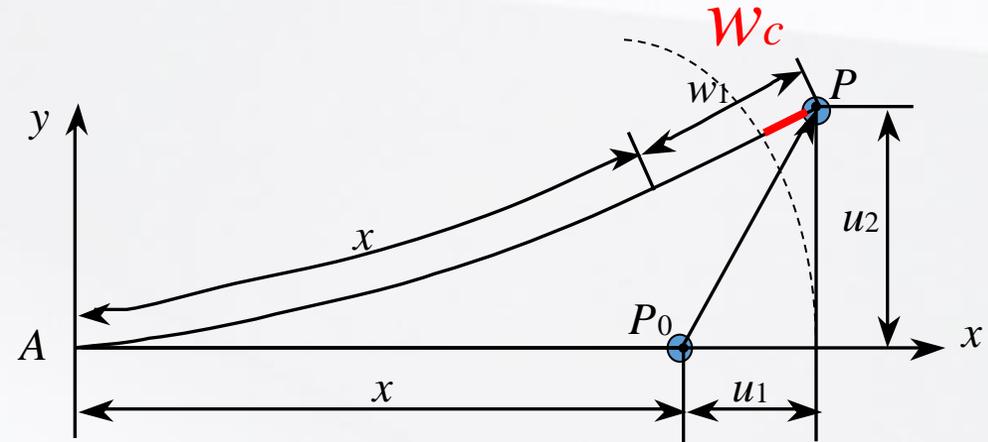
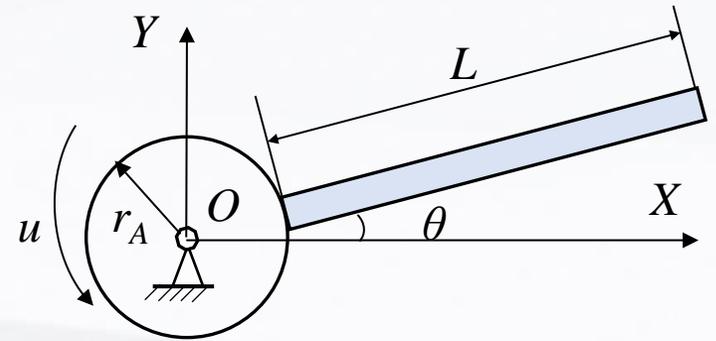
点P变形: $\mathbf{r}_P = \overrightarrow{OP} = \mathbf{r}_A + \Theta(\mathbf{r}_0 + \mathbf{r}_1)$

$$\mathbf{r}_0 = \overrightarrow{OP_0} \quad \mathbf{r}_1 = \overrightarrow{P_0P}$$

$$\Theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

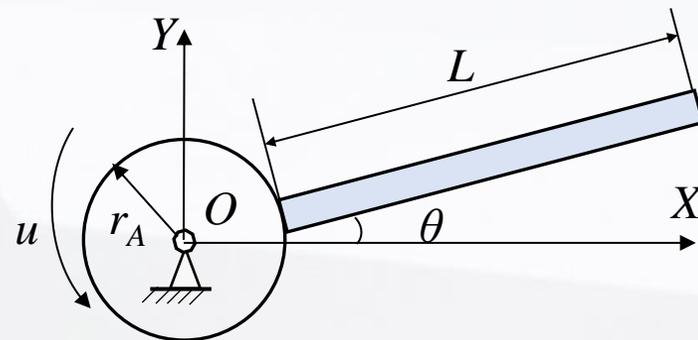
$\mathbf{r}_1 = \overrightarrow{P_0P}$ 为 P_0 点的变形位移矢量, 坐标为 $[u_1(x,t), u_2(x,t)]^T$

$$\mathbf{r}_1 = \begin{bmatrix} u_1(x,t) \\ u_2(x,t) \end{bmatrix} = \begin{bmatrix} w_1(x,t) + w_c(x,t) \\ w_2(x,t) \end{bmatrix} = \begin{bmatrix} w_1(x,t) - \frac{1}{2} \int_0^x \left(\frac{\partial w_2(\xi,t)}{\partial \xi} \right)^2 d\xi \\ w_2(x,t) \end{bmatrix}$$



$$\mathbf{r}_P = \overline{OP} = \mathbf{r}_A + \Theta(\mathbf{r}_0 + \mathbf{r}_1)$$

$$\mathbf{r}_1 = \begin{bmatrix} w_1(x,t) - \frac{1}{2} \int_0^x \left(\frac{\partial w_2(\xi,t)}{\partial \xi} \right)^2 d\xi \\ w_2(x,t) \end{bmatrix}$$



系统动能: $T = \frac{1}{2} J_H \dot{\theta}^2 + \frac{1}{2} \int_0^L \rho A \dot{\mathbf{r}}_P^T \dot{\mathbf{r}}_P dx$

系统势能: $H = \frac{1}{2} \int_0^L EA [w_1'(x,t)]^2 dx + \frac{1}{2} \int_0^L EI [w_2''(x,t)]^2 dx$

外力虚功: $\delta W_F = u \delta \theta$

Hamilton变分原理: $\int_{t_1}^{t_2} (\delta T - \delta H + \delta W_F) dt = 0$

$$J_H \ddot{\theta} + \int_0^l \rho A \left\{ \ddot{\theta} \left[(r_A + x + w_1 + w_c)^2 + w_2^2 \right] + 2\dot{\theta} \left[(r_A + x + w_1 + w_c)(\dot{w}_1 + \dot{w}_c) + w_2 \dot{w}_2 \right] + (r_A + x + w_1 + w_c) \ddot{w}_2 - w_2 (\ddot{w}_1 + \ddot{w}_c) \right\} dx = u(t)$$

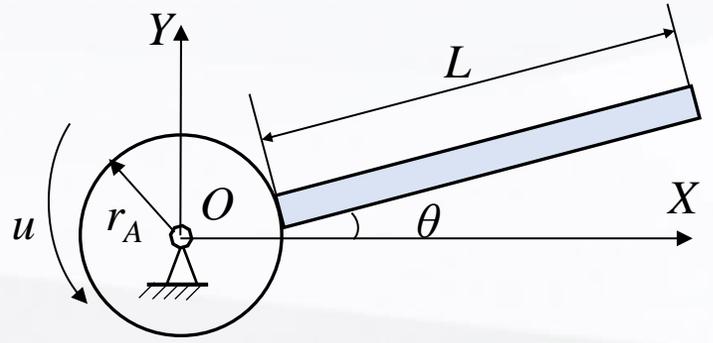
$$\int_0^l \left[\rho A (\ddot{w}_1 + \ddot{w}_c) - 2\rho A \dot{\theta} \dot{w}_2 - \rho A \ddot{\theta} w_2 - \rho A \dot{\theta}^2 (r_A + x + w_1 + w_c) - EA w_1'' \right] dx = 0$$

$$\int_0^l \left[\rho A \ddot{w}_2 + 2\rho A \dot{\theta} (\dot{w}_1 + \dot{w}_c) + \rho A \ddot{\theta} (r_A + x + w_1 + w_c) - \rho A \dot{\theta}^2 w_2 + EI w_2'''' + \rho A \frac{\partial}{\partial x} \left(w_2' \int_x^l B(\xi, t) d\xi \right) \right] dx = 0$$

假设模态离散: $w_1(x,t) = \Phi_1(x)\eta_1(t)$, $w_2(x,t) = \Phi_2(x)\eta_2(t)$

$\Phi_1(x) \in R^{1 \times n}$ $\Phi_2(x) \in R^{1 \times n}$ 模态函数

$\eta_1(t) \in R^n$ $\eta_2(t) \in R^n$ 模态坐标



耦合位移场混合坐标模型:

$$\begin{bmatrix} J_H + M_{\theta\theta} & M_{\theta\eta_1} & M_{\theta\eta_2} \\ M_{\eta_1\theta} & M_{\eta_1\eta_1} & 0 \\ M_{\eta_2\theta} & 0 & M_{\eta_2\eta_2} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\eta}_1 \\ \ddot{\eta}_2 \end{bmatrix} + 2\dot{\theta} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & G_{\eta_1\eta_2} \\ 0 & G_{\eta_2\eta_1} & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_{\eta_1\eta_1} & 0 \\ 0 & 0 & K_{\eta_2\eta_2} \end{bmatrix} \begin{bmatrix} \theta \\ \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} q_\theta \\ q_{\eta_1} \\ 0 \end{bmatrix} + \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}$$

$$M\ddot{x} + 2\dot{\theta}G\dot{x} + Kx = q + f$$

$$x = \begin{bmatrix} \theta \\ \eta_1 \\ \eta_2 \end{bmatrix} \quad M = \begin{bmatrix} J_H + M_{\theta\theta} & M_{\theta\eta_1} & M_{\theta\eta_2} \\ M_{\eta_1\theta} & M_{\eta_1\eta_1} & 0 \\ M_{\eta_2\theta} & 0 & M_{\eta_2\eta_2} \end{bmatrix} \quad G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & G_{\eta_1\eta_2} \\ 0 & G_{\eta_2\eta_1} & 0 \end{bmatrix} \quad K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_{\eta_1\eta_1} & 0 \\ 0 & 0 & K_{\eta_2\eta_2} \end{bmatrix} \quad q = \begin{bmatrix} q_\theta \\ q_{\eta_1} \\ 0 \end{bmatrix} \quad f = \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}$$

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$$\begin{bmatrix} J_H + M_{\theta\theta} & M_{\theta\eta_1} & M_{\theta\eta_2} \\ M_{\eta_1\theta} & M_{\eta_1\eta_1} & \mathbf{0} \\ M_{\eta_2\theta} & \mathbf{0} & M_{\eta_2\eta_2} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\eta}_1 \\ \ddot{\eta}_2 \end{bmatrix} + 2\dot{\theta} \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{G}_{\eta_1\eta_2} \\ \mathbf{0} & \mathbf{G}_{\eta_2\eta_1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{\eta_1\eta_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{\eta_2\eta_2} \end{bmatrix} \begin{bmatrix} \theta \\ \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} q_\theta \\ \mathbf{q}_{\eta_1} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} u \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

$\theta \in \mathcal{R}^{1 \times 1}$ 系统大范围运动角位移

$\eta_1 \in R^n$ $\eta_2 \in R^n$ 梁轴向和横向振动模态坐标列向量

J_H 中心刚体转动惯量 $M_{\theta\theta} \in \mathcal{R}^{1 \times 1}$ 梁转动惯量

$M_{\eta_1\eta_1} \in \mathcal{R}^{n \times n}$ $M_{\eta_2\eta_2} \in \mathcal{R}^{n \times n}$ 梁轴向和横向振动广义弹性质量阵

$M_{\theta\eta_1} \in R^{1 \times n}$ $M_{\theta\eta_2} \in R^{1 \times n}$ $M_{\eta_1\theta} \in R^{n \times 1}$ $M_{\eta_2\theta} \in R^{n \times 1}$

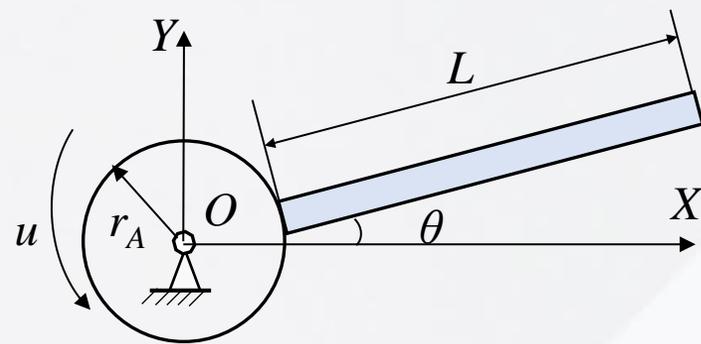
代表着大范围运动和柔性梁弹性变形之间的非线性惯性耦合

$\mathbf{G}_{\eta_1\eta_2} \in R^{n \times n}$ $\mathbf{G}_{\eta_2\eta_1} \in R^{n \times n}$ 来源于陀螺效应

$\mathbf{K}_{\eta_1\eta_1} \in R^{n \times n}$ $\mathbf{K}_{\eta_2\eta_2} \in R^{n \times n}$ 刚度阵

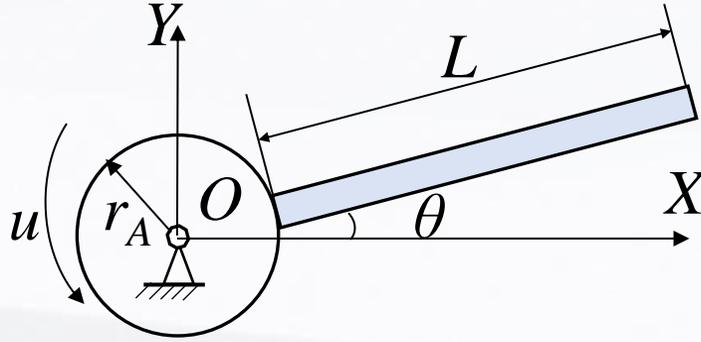
$q_\theta \in R^1$ $\mathbf{q}_{\eta_1} \in R^{n \times 1}$ 惯性力项

$u(t) \in \mathcal{R}^{1 \times 1}$ 中心刚体处控制力矩



耦合位移场混合坐标模型

$$M\ddot{x} + 2\dot{\theta}G\dot{x} + Kx = q + f$$



考虑阻尼 总阻尼矩阵 :

$$C_t = \begin{bmatrix} C_H & 0 & 0 \\ 0 & \alpha_1 M_1 & 0 \\ 0 & 0 & \alpha_2 M_2 \end{bmatrix} + \frac{\beta_1}{\rho A} \begin{bmatrix} M_{\theta\theta} & M_{\theta\eta_1} & M_{\theta\eta_2} \\ M_{\eta_1\theta} & M_{\eta_1\eta_1} & 0 \\ M_{\eta_2\theta} & 0 & M_{\eta_2\eta_2} \end{bmatrix} + \frac{\beta_2 \dot{\theta} \text{sign}(\dot{\theta})}{\rho A} \begin{bmatrix} C_{11} & C_{21}^T & C_{31}^T \\ C_{21} & C_{22} & 0 \\ C_{31} & 0 & C_{33} \end{bmatrix}$$

轴承阻尼

结构阻尼

线性空气阻尼阵

$$\vec{F}_1^A = -\beta_1 \dot{r}_P$$

平方空气阻尼阵

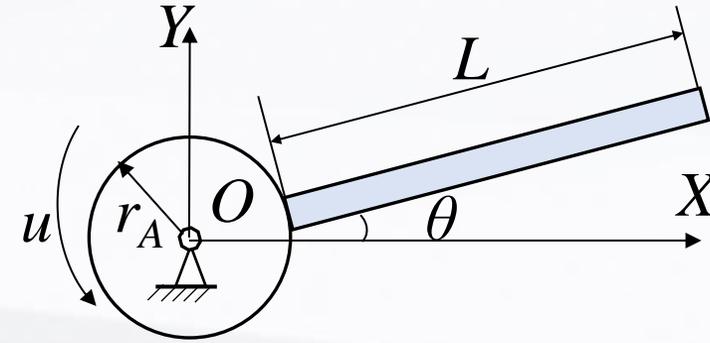
$$\vec{F}_2^A = -\beta_2 \dot{r}_P \left| \dot{r}_P \right|$$

已有研究结果显示:

物体低速运动时线性空气阻尼占主要成分, 度高速时平方阻尼占主要成分

耦合位移场混合坐标模型

$$M\ddot{x} + 2\dot{\theta}G\dot{x} + Kx = q + f$$



考虑阻尼 总阻尼矩阵 :

$$C_t = \begin{bmatrix} C_H & 0 & 0 \\ 0 & \alpha_1 M_1 & 0 \\ 0 & 0 & \alpha_2 M_2 \end{bmatrix} + \frac{\beta_1}{\rho A} \begin{bmatrix} M_{\theta\theta} & M_{\theta\eta_1} & M_{\theta\eta_2} \\ M_{\eta_1\theta} & M_{\eta_1\eta_1} & 0 \\ M_{\eta_2\theta} & 0 & M_{\eta_2\eta_2} \end{bmatrix} + \frac{\beta_2 \dot{\theta} \text{sign}(\dot{\theta})}{\rho A} \begin{bmatrix} C_{11} & C_{21}^T & C_{31}^T \\ C_{21} & C_{22} & 0 \\ C_{31} & 0 & C_{33} \end{bmatrix}$$

轴承阻尼 结构阻尼

线性空气阻尼阵

$$\vec{F}_1^A = -\beta_1 \dot{\vec{r}}_P$$

平方空气阻尼阵

$$\vec{F}_2^A = -\beta_2 \dot{\vec{r}}_P \left| \dot{\vec{r}}_P \right|$$

考虑阻尼的耦合位移场混合坐标模型:

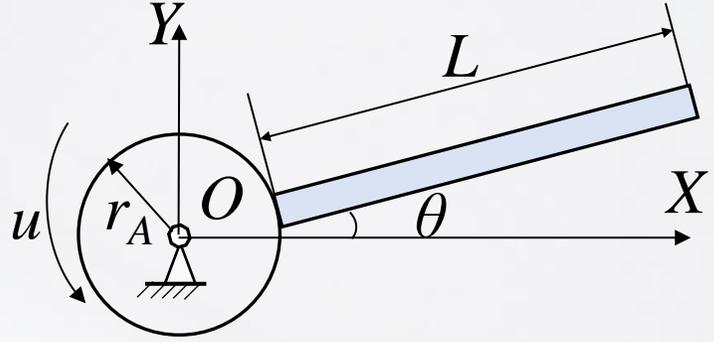
$$M\ddot{x} + (2\dot{\theta}G + C_t)\dot{x} + Kx = q + f$$

考虑阻尼的耦合位移场混合坐标模型：

$$M\ddot{x} + (2\dot{\theta}G + C_t)\dot{x} + Kx = q + f$$

$$Y = \begin{bmatrix} \theta \\ \eta_1 \\ \eta_2 \end{bmatrix} \quad M = \begin{bmatrix} J_H + M_{\theta\theta} & M_{\theta\eta_1} & M_{\theta\eta_2} \\ M_{\eta_1\theta} & M_{\eta_1\eta_1} & 0 \\ M_{\eta_2\theta} & 0 & M_{\eta_2\eta_2} \end{bmatrix} \quad G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & G_{\eta_1\eta_2} \\ 0 & G_{\eta_2\eta_1} & 0 \end{bmatrix} \quad K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_{\eta_1\eta_1} & 0 \\ 0 & 0 & K_{\eta_2\eta_2} \end{bmatrix} \quad q = \begin{bmatrix} q_\theta \\ Q_{\eta_1} \\ 0 \end{bmatrix} \quad f = \begin{bmatrix} \tau \\ 0 \\ 0 \end{bmatrix}$$

与横向变形相比梁轴向变形很小，对系统动力学影响很小



耦合位移场混合坐标简化模型：

$$\begin{bmatrix} J_H + M_{\theta\theta} & M_{\theta\eta_2} \\ M_{\eta_2\theta} & M_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\eta}_2 \end{bmatrix} + \begin{bmatrix} \tilde{C}_{11} & \tilde{C}_{12} \\ \tilde{C}_{12}^T & \tilde{C}_{22} \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\eta}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_{\eta_2\eta_2} \end{bmatrix} \begin{bmatrix} \theta \\ \eta_2 \end{bmatrix} = \begin{bmatrix} q_\theta \\ 0 \end{bmatrix} + \begin{bmatrix} u \\ 0 \end{bmatrix}$$

以下描述参数改动： $\eta_2 \rightarrow \eta$ $M_2 \rightarrow M$

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$$\begin{bmatrix} J_H + M_{\theta\theta} & \mathbf{M}_{\theta\eta} \\ \mathbf{M}_{\eta\theta} & \mathbf{M} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\boldsymbol{\eta}} \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{C}}_{11} & \tilde{\mathbf{C}}_{12} \\ \tilde{\mathbf{C}}_{12}^T & \tilde{\mathbf{C}}_{22} \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\boldsymbol{\eta}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{\eta\eta} \end{bmatrix} \begin{bmatrix} \theta \\ \boldsymbol{\eta} \end{bmatrix} = \begin{bmatrix} q_\theta \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} u \\ \mathbf{0} \end{bmatrix}$$

$$M_{\theta\theta} = J_1 + \boldsymbol{\eta}^T \mathbf{M} \boldsymbol{\eta} - \boldsymbol{\eta}^T (r_A \mathbf{D}_0 + \mathbf{D}_1) \boldsymbol{\eta} \quad \mathbf{M}_{\theta\eta} = \mathbf{M}_{\eta\theta}^T = r_A \mathbf{U}_{02} + \mathbf{U}_{12}$$

$$\mathbf{M} = \int_0^L \rho A \boldsymbol{\Phi}^T \boldsymbol{\Phi} dx \quad q_\theta = -2\dot{\theta} [\boldsymbol{\eta}^T \mathbf{M} \dot{\boldsymbol{\eta}} - \boldsymbol{\eta}^T (r_A \mathbf{D}_0 + \mathbf{D}_1) \dot{\boldsymbol{\eta}}]$$

$$\mathbf{K}_{\eta\eta} = \mathbf{K} - \dot{\theta}^2 \mathbf{M} + \dot{\theta}^2 (r_A \mathbf{D}_0 + \mathbf{D}_1)$$

$$\tilde{\mathbf{C}}_{11} = C_H + \frac{\beta_1}{\rho A} M_{\theta\theta} + \frac{\beta_2 \dot{\theta} \text{sign}(\dot{\theta})}{\rho A} \left[C_J + \boldsymbol{\eta}^T (r_A \mathbf{M} + \mathbf{U}_{13}) \boldsymbol{\eta} - \boldsymbol{\eta}^T (r_A^2 \mathbf{D}_0 + 2r_A \mathbf{D}_1 + \mathbf{D}_2) \boldsymbol{\eta} \right]$$

$$\tilde{\mathbf{C}}_{12} = \frac{\beta_1}{\rho A} \mathbf{M}_{\theta\eta} + \frac{\beta_2 \dot{\theta} \text{sign}(\dot{\theta})}{\rho A} \left[r_A^2 \mathbf{U}_{02}^T + 2r_A \mathbf{U}_{12}^T + \int_0^L \rho A x^2 \boldsymbol{\Phi}^T dx \right]^T$$

$$\tilde{\mathbf{C}}_{22} = \left(\alpha + \frac{\beta_1}{\rho A} \right) \mathbf{M} + \frac{\beta_2 \dot{\theta} \text{sign}(\dot{\theta})}{\rho A} \left[r_A \mathbf{M} + \int_0^L \rho A x \boldsymbol{\Phi}^T \boldsymbol{\Phi} dx \right]$$

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