CHAPTER 3

Lightwave Fundamentals

Wave propagation is important in fiber optics. In this chapter, we present fundamental aspects of wave travel that are particularly valuable. For some reason, the prospect of studying electromagnetic waves frightens many people. Indeed, expositions of electromagnetic theory are often quite formidable. The discussion to follow is as cheerful and painless as possible. Important results are explained, but the lengthy derivations required to develop them are omitted. Mathematics is minimized. The specific concepts developed are velocity, power, dispersion, polarization, interference, and reflections at boundaries. All of these relate directly to fiber optics.

3.1 ELECTROMAGNETIC WAVES

Light consists of an electric field and a magnetic field that oscillate at very high rates (on the order of 10^{14} Hz). These fields travel in wavelike fashion at very high speeds. A picture of an electromagnetic wave [1] traveling along the z direction appears in Fig. 3.1. The electric field is plotted at three times, showing the progress of the wave. At any fixed location, the field amplitude varies at the optic frequency. The amplitude repeats itself after one period of the oscillation. The wave repeats itself in space, at a fixed time, after a distance λ. This distance is the *wavelength*. Its reciprocal, $1/\lambda$, is the *wave number*.

The electric field for the wave sketched in Fig. 3.1 can be written as

$$E = E_o \sin(\omega t - kz) \qquad (3.1)$$

where E_o is the peak amplitude, $\omega = 2\pi f$ rad/s, and f is the frequency in hertz. The factor ω is called the *radian frequency*. The term k is the *propagation factor*. It is given by

$$k = \frac{\omega}{v} \qquad (3.2)$$

where v is the phase velocity of the wave. The factor $\omega t - kz$ is the *phase* of the wave, and kz is the *phase shift* due to travel over length z. A *plane wave* is one whose phase is the same over a planar surface. In the present example, the phase is the same over

FIGURE 3.1

Electric field for a wave traveling in the z direction. The field is drawn at three different times to illustrate the motion of the wave in the direction of travel.

any plane defined by a fixed value of z, so that Eq. (3.1) represents a plane wave. If time is held constant, then Eq. (3.1) shows the sinusoidal spatial variation of the field. For example, if $t = 0$, then $E = E_o \sin(-kz) = -E_o \sin kz$. On the other hand, if the position is fixed, then Eq. (3.1) shows the sinusoidal time variation of the field. Taking the fixed position as the origin, $z = 0$, yields $E = E_o \sin \omega t$, illustrating this point.

In terms of the refractive index n, the velocity is $v = c/n$, so that

$$k = \frac{\omega n}{c} \tag{3.3}$$

The propagation constant in free space will be denoted by k_0. Because $n = 1$ in free space,

$$k_0 = \frac{\omega}{c} \tag{3.4}$$

Combining Eqs. (3.3) and (3.4) shows that the propagation constant in any medium can be given in terms of the free-space propagation value by

$$k = k_0 n \tag{3.5}$$

According to Eq (1.3), $\lambda = v/f$. Substituting this into Eq. (3.2) yields

$$k = \frac{2\pi}{\lambda} \tag{3.6}$$

This equation relates the propagation constant in a medium to the wavelength in that medium. The free-space wavelength is $\lambda_0 = c/f$, and the wavelength in any medium is $\lambda = v/f$; hence,

$$\frac{\lambda_0}{\lambda} = \frac{c}{v} = n \tag{3.7}$$

The wavelength in a medium is shorter than in free space, because the refractive index is greater than unity.

The power in an optic beam is proportional to the light *intensity* (defined as the square of the electric field). Intensity is proportional to *irradiance,* the power density.

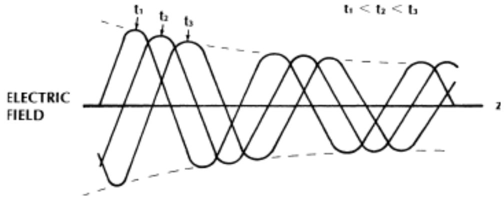

FIGURE 3.2
Attenuation of a traveling wave.

The units of irradiance are watts per square meter. In Section 2.5, we investigated the intensity variation of a particular light distribution, the Gaussian beam. Sometimes intensity is used to describe the total power in a wave. This use, although not accurate, is common.

If a wave does not lose energy as it propagates, then Eq. (3.1) and Fig. 3.1 provide appropriate descriptions. If attenuation is important, than the equation and the figure must be modified. The corrected equation is

$$E = E_o e^{-\alpha z} \sin(\omega t - kz) \tag{3.8}$$

where ω and k have the same meaning as in Eq. (3.1). The term α is the *attenuation coefficient.* (In a fiber, it represents the losses in the fiber.) Its value describes the rate at which the electric field diminishes as it travels through the lossy medium. Although the decay is exponential, the attenuation coefficient is so small for quality fibers that there is little attenuation (maybe just a few decibels) even over long paths. In a lossy medium, the field appears as shown in Fig. 3.2. The dashed line on the figure is a curve of the factor $\exp(-\alpha z)$, describing the loss in Eq. (3.8).

The intensity of a light beam is proportional to the square of its electric field. Therefore, the power in the beam corresponding to Eq. (3.8) diminishes as $\exp(-2\alpha z)$. For a path of length L, the ratio of the output power to the input power is $\exp(-2\alpha L)$. The power reduction in decibels is thus

$$\text{dB} = 10 \log_{10} \exp(-2\alpha L)$$

This will turn out to be a negative number for propagation through a lossy medium. From this last expression, we can find the relationship between the attenuation coefficient and the power change in dB/km (γ). The result is

$$\gamma = -8.685\alpha, \ \text{dB/km}$$

where α is given in units of km^{-1}. (We ask the student to prove this relationship in Problem 3.14.) Still another useful relationship between the input and output powers and the transmission loss is the following, known as *Beer's law:*

$$P_{\text{out}}/P_{\text{in}} = 10^{\gamma L/10}$$