Hexagonal Grating

Introduction

A hexagonal grating is an infinite structure that is periodic with hexagonal (or rhomboid unit) cells. Figure 1 shows the hexagonal domain used for this model. The reflecting perfectly conducting surface consists of regularly spaced protruding semispheres.

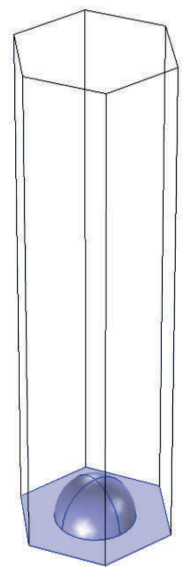

Figure 1: The hexagonal domain, used for computing the diffraction from the hexagonal grating.

As shown in Figure 2, for a hexagonal cell of side length a, the corresponding unit cell is a rhomboid with side length $\sqrt{3}a$. In Figure 2, the side vectors for the hexagonal cell starts from the point P and are denoted $\mathbf{a}_1$ and $\mathbf{a}_2$. The angle between $\mathbf{a}_1$ and $\mathbf{a}_2$ is 120 degrees. Similarly, for the rhomboid unit cell, the primitive vectors are denoted

$\mathbf{u}_1$ and $\mathbf{u}_2$ and starts from the hexagon center point Q. The angle between the two primitive vectors is also 120 degrees.

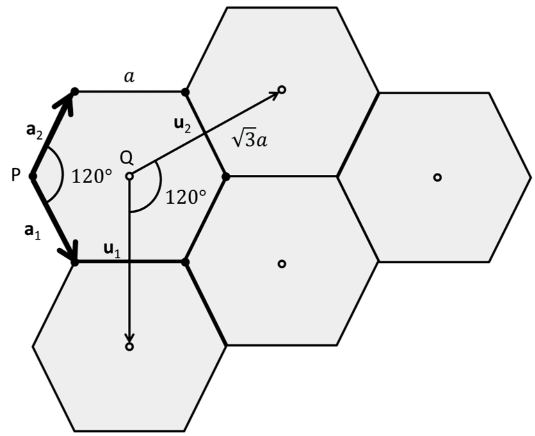

Figure 2: Schematic showing the hexagonal cells with side length a *and side vectors* $\mathbf{a}_1$ *and* $\mathbf{a}_2$*. The primitive cells are defined by the primitive vectors* $\mathbf{u}_1$ *and* $\mathbf{u}_2$*.*

If the incident plane wave have a wave vector defined by

$$\mathbf{k} = \mathbf{k}_{||} + \mathbf{k}_{\perp}, \tag{1}$$

where $\mathbf{k}_{||}$ is the wave vector component parallel to the periodic boundary and $\mathbf{k}\perp$ is the component orthogonal to the periodic boundary, the in-plane wave vector component for diffraction order mn is given by

$$\mathbf{k}_{||mn} = \mathbf{k}_{||} + m\mathbf{G}_1 + n\mathbf{G}_2, \tag{2}$$

where the reciprocal lattice vectors $\mathbf{G}_1$ and $\mathbf{G}_2$ are defined from the primitive vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ as

$$\mathbf{G}_1 = 2\pi\frac{\mathbf{u}_2 \times \mathbf{n}}{\mathbf{u}_1 \cdot (\mathbf{u}_2 \times \mathbf{n})} \tag{3}$$

and

$$\mathbf{G}_2 = 2\pi\frac{\mathbf{n} \times \mathbf{u}_1}{\mathbf{u}_2 \cdot (\mathbf{n} \times \mathbf{u}_1)}, \tag{4}$$

where $\mathbf{n}$ is the normal vector (length 1) to the periodic boundary.

Since the out-of-plane wave vector component for mode mn is defined by

$$k_{\perp mn} = \sqrt{k^2 - k^2_{||mn}}, \tag{5}$$

it is clear that for propagating modes, where the out-of-plane wave vector component above must be real, the length of the in-plane wave vector component must be smaller than the material wave number k. Figure 3 shows that it is only the modes inside the circle with radius k that will be propagating. In the example shown in Figure 3, there are five modes that will be propagating, in this case the modes $m = n = 0$ (the reflected wave), $m = -1$, $n = 0$, $m = 0$, $n = -1$, $m = -1$, $n = -1$, and $m = -2$, $n = -1$. All other modes will be evanescent and damped out.

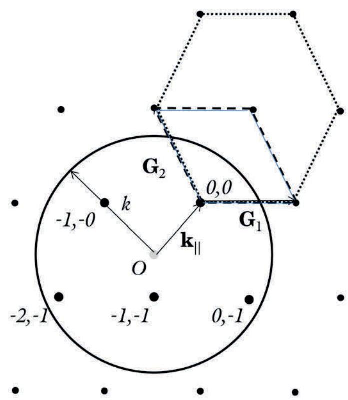

Figure 3: The reciprocal lattice, showing the reciprocal lattice vectors $\mathbf{G}_1$ *and* $\mathbf{G}_2$*, the in-plane wave vector component* $\mathbf{k}_{||}$*, and the circle with radius* k *(the material wave number) enclosing the propagating mode points (larger dots. The dotted hexagon indicates that also the reciprocal lattice is a hexagonal point lattice. The dashed rhomboid indicates the unit cell spanned by the reciprocal lattice vectors.*

Model Definition

In this model, the unit cell is small compared to the wavelength, so there will only be two modes that are propagating, the modes $m = 0$, $n = -1$ and $m = -1$, $n = -1$. For wavelengths longer than approximately 1.01 μm (the critical wavelength), the mode $m = 0$, $n = -1$ will be evanescent.

First a wavelength sweep will be made for an incident field having the polarization perpendicular to the plane of incidence (spanned by the wave vector for the incident wave and the normal to the periodic boundary) (so called s-polarization). Thereafter another wavelength sweep is made, but now with the polarization in the plane of incidence (p-polarization).

Results and Discussion

Figure 4 shows the electric field norm and the propagation directions for the incident, the reflected and the diffracted waves. Notice that the diffracted waves come in pairs (both have the same mode numbers), one wave having the polarization in the plane-of-diffraction and the other wave have orthogonal polarization to the plane-of-diffraction. The plane-of-diffraction is spanned by the wave vector for the diffracted wave and the normal to the periodic boundary. The wavelength is close the critical wavelength for the $m = 0$, $n = -1$ mode. This is evident from the plot, as the wave vector for that mode (the yellow arrows) is almost parallel to the periodic boundary.

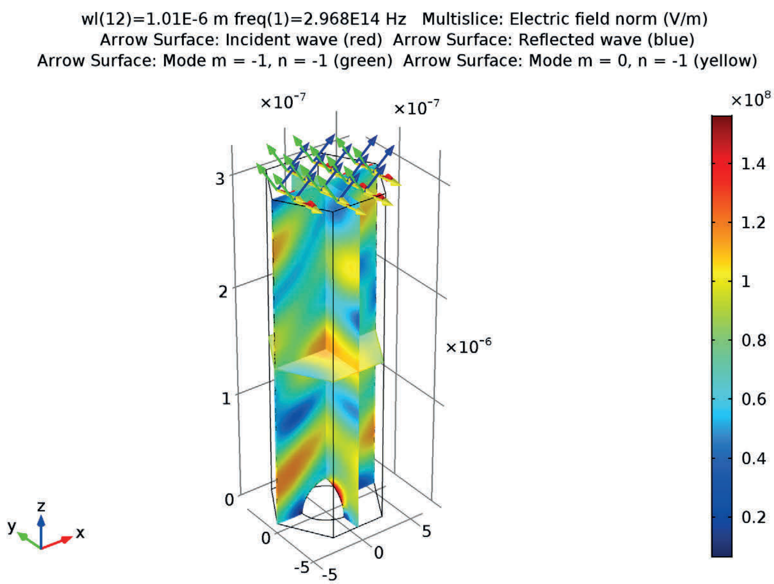

Figure 4: The electric field norm and the propagation directions for the incident wave (red arrows), the reflected wave (blue arrows) and the two diffraction orders (green and yellow arrows). The wavelength is 1.01 µm, which is close to the critical wavelength for the mode $m = 0$, $n = -1$, and the polarization of the incident wave is perpendicular to the plane of incidence.

Figure 5 shows the reflectance (for mode $m = n = 0$) and the diffraction efficiencies for the diffracted waves. Notice that both the reflectance and the diffraction efficiency for the in-plane-polarized m = -1, n = -1 mode show resonances (peaks or dips) close to the critical wavelength for the m = 0, n = -1 modes.

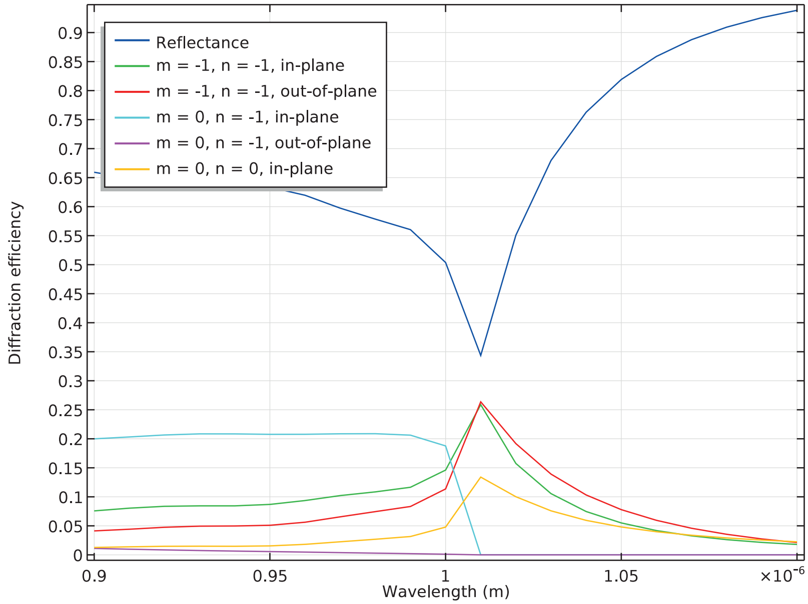

Figure 5: Diffraction efficiencies for the reflected wave and the diffracted waves. The polarization of the incident wave is perpendicular to the plane of incidence.

Figure 6 shows a similar plot as Figure 4, but here the polarization of the incident wave is parallel with the plane of incidence.

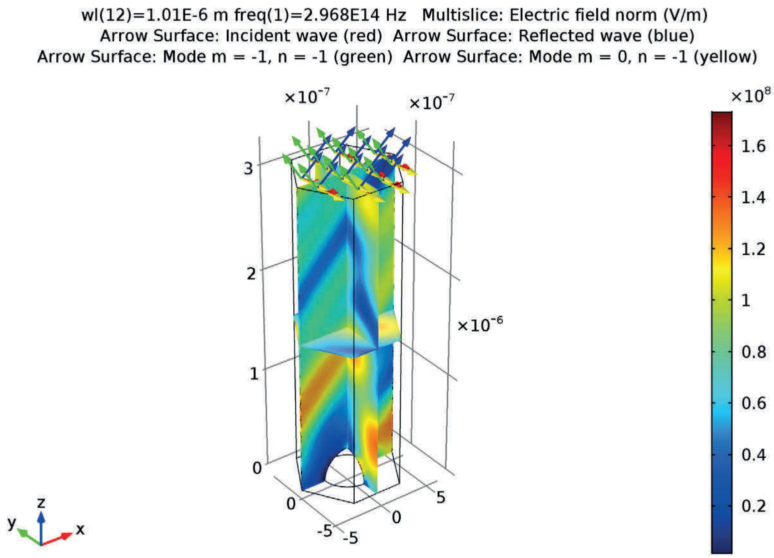

Figure 6: Similar plot as in Figure 4*, but here the polarization of the incident wave is parallel to the plane of incidence.*