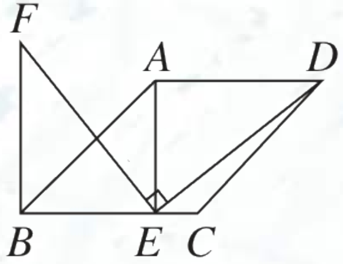
阶段拔尖专训12 特殊四边形中的类比探究

题型1 平行四边形中的类比探究

1.**【问题背景】**在▱$ABCD$中,$AE\perp BC$,垂足为E,连结DE,将ED绕点E逆时针旋转90°,得到EF,连结BF.



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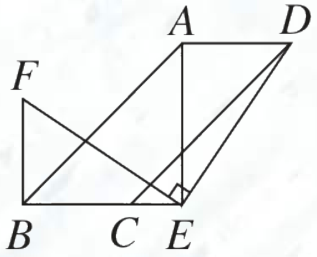

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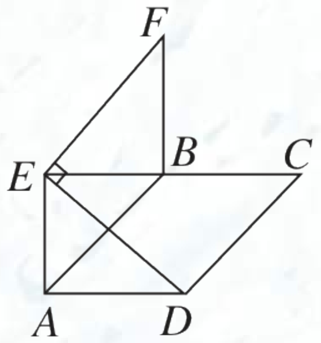

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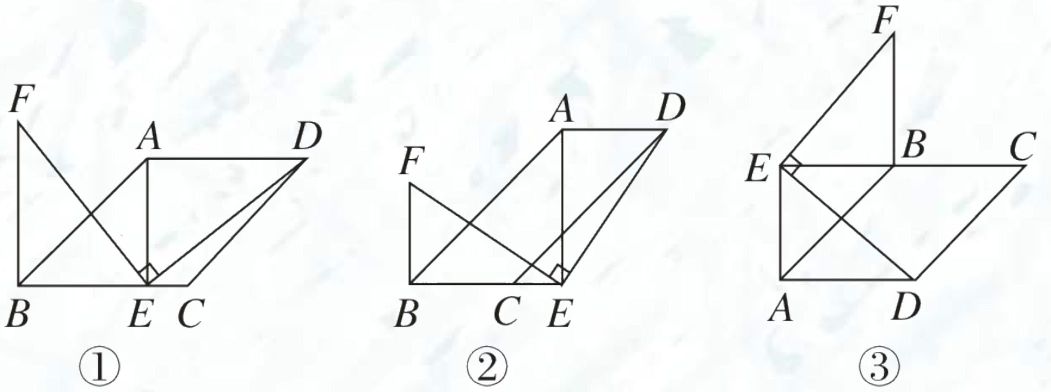
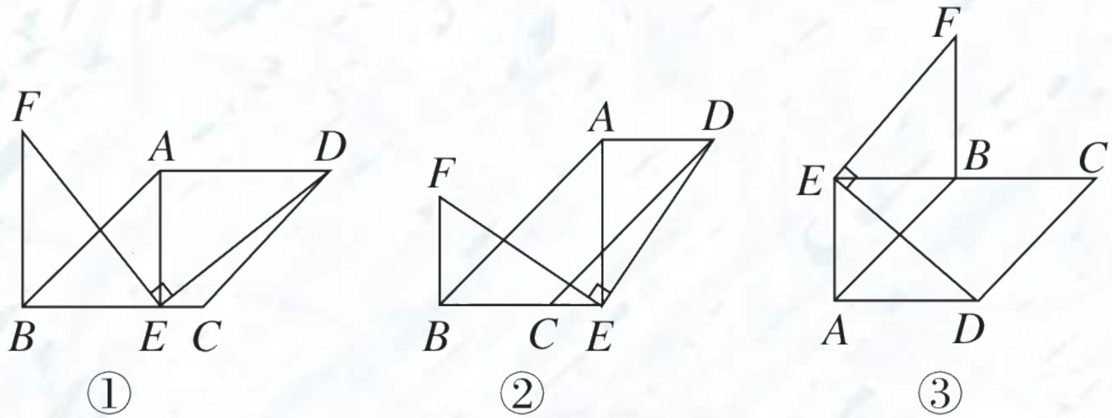
【问题发现】 如图①，当点E在线段BC上，$\angle ABC = 45^\circ$ 时，请直接写出线段AE，EC，BF的数量关系；



【解】$AE + EC = BF$.

【问题探究】 如图②,当点E在线段BC的延长线上,且$\angle ABC = 45^\circ$时，请猜想线段AE，EC，BF的数量关系，并说明理由；



$AE-EC=BF$.理由如下:

$\because AE\perp BC$交BC的延长线于点E,$\therefore \angle AEB=90^\circ$.

又$\because \angle ABC=45^\circ$,$\therefore \angle BAE=45^\circ=\angle ABC$,$\therefore BE=AE$.

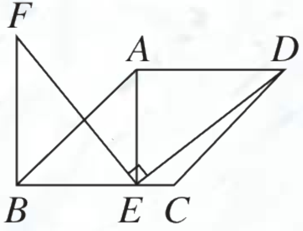
$\because$ 将ED绕点E逆时针旋转90° ,得到EF,$\therefore \angle DEF=90^\circ$,

$EF=ED$,

$\therefore \angle AED=90^\circ-\angle AEF=\angle BEF$. $\therefore \triangle BEF\cong\triangle AED$,

$\therefore BF=AD$. $\because$ 四边形$ABCD$是平行四边形,

$\therefore BC = AD$，$\therefore AE - EC = BE - EC = BC = AD = BF$，即 $AE - EC = BF$.



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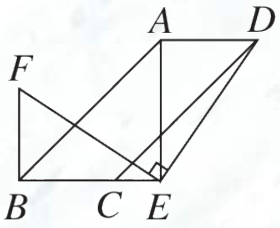

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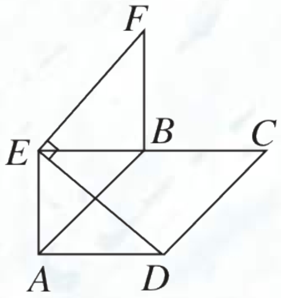

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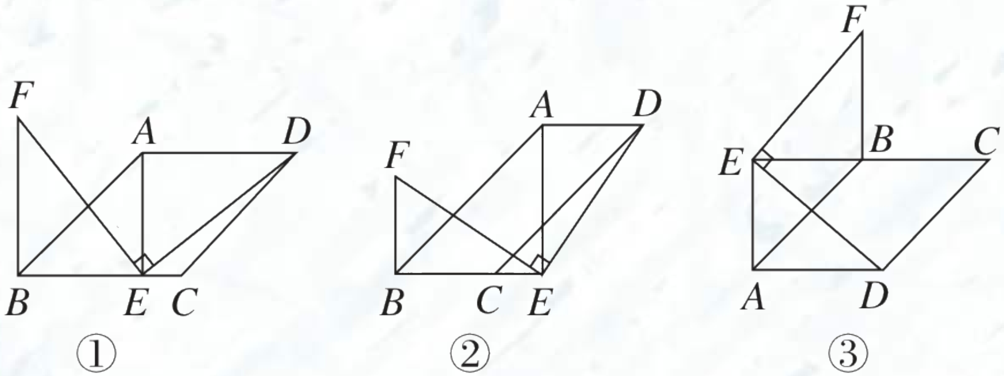
【问题拓展】 如图③，当点E在线段CB的延长线上，且$\angle ABC = 135°$时，请猜想线段AE，EC，BF的数量关系，并说明理由.



$EC-AE=BF$.理由如下:

$\because AE\perp BC$交CB的延长线于点E，$\therefore \angle AEB=90°$.

又$\because \angle ABC=135°$ ，$\therefore \angle ABE=180°-\angle ABC=45°$ ，

$\therefore \angle BAE=45°=\angle ABE$，$\therefore BE=AE$.

$\because$ 将ED绕点E逆时针旋转$90°$ ，得到EF,

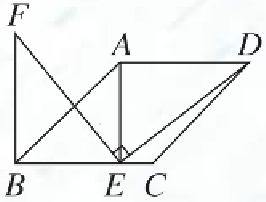
$\therefore \angle DEF=90°$ ，$EF=ED$,

$\therefore \angle BEF=90°-\angle BED=\angle AED$,

$\therefore \triangle BEF \cong \triangle AED, \therefore BF = AD.$

$\because$ 四边形$ABCD$是平行四边形，$\therefore BC = AD,$

$\therefore EC - AE = EC - BE = BC = AD = BF$，即$EC - AE = BF.$



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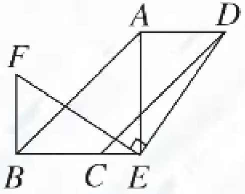

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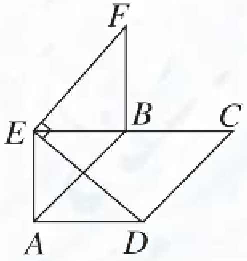

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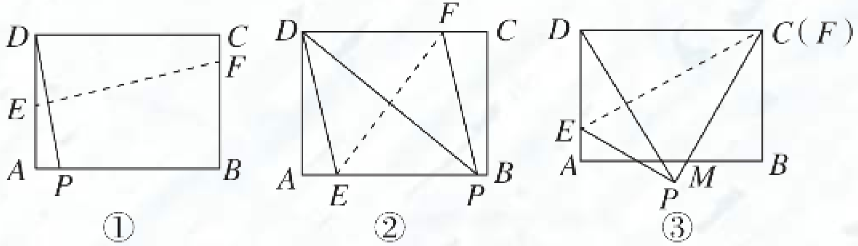
题型2 矩形中的类比探究

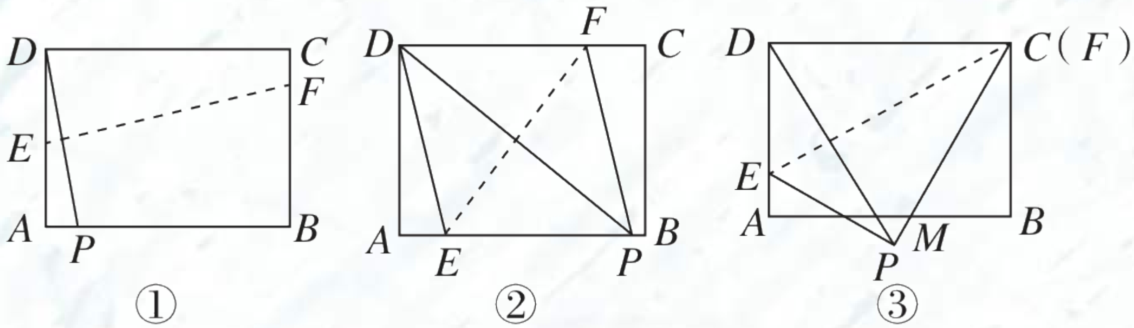
2.[2024扬州期中] **【实践操作】**在矩形$ABCD$中，$AB=4$，$AD=3$，现将纸片折叠，点D的对应点记为点P，折痕为EF（点E，F是折痕与矩形的边的交点），再将纸片还原，连结DP.



【初步思考】

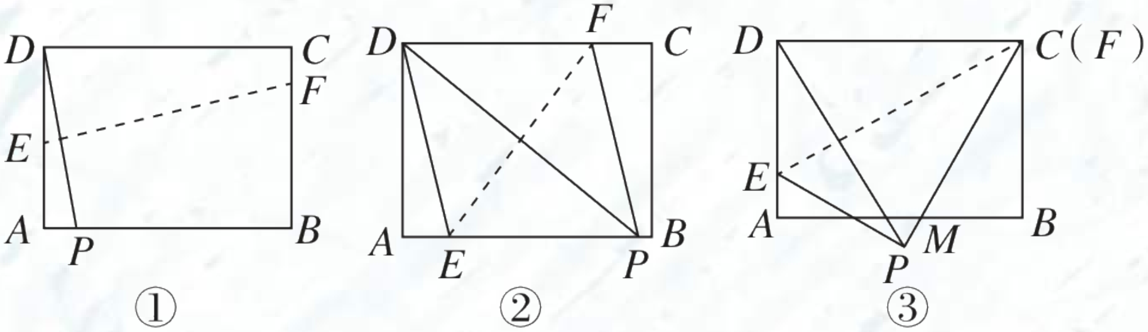
（1）若点P落在矩形$ABCD$的边AB上（如图①）.

当点P与点A重合时，$\angle DEF =$ <u>90</u> °；当点E与点A重合时，$\angle DEF =$ <u>45</u> °；



【深入探究】

(2) 当点E在AB上，点F在DC上时（如图②），连结DE，PF，试说明：四边形$DEPF$为菱形，并直接写出当$AP=\frac{7}{2}$时菱形$DEPF$的边长.



【解】设EF与DP相交于点O.

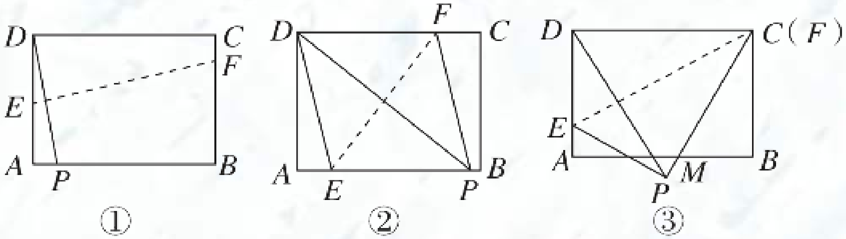
$\because$ 由题意得EF是PD的中垂线，$\therefore DO=PO$，$EF\perp PD$. $\because$ 四边形$ABCD$是矩形，$\therefore DC//AB$，$\therefore \angle FDO=\angle EPO$.又

$\because \angle DOF=\angle EOP$，$\therefore \triangle DOF\cong\triangle POE$，$\therefore DF=PE$.又

$\because DF//PE$，$\therefore$ 四边形$DEPF$是平行四边形.又$\because EF\perp PD$，

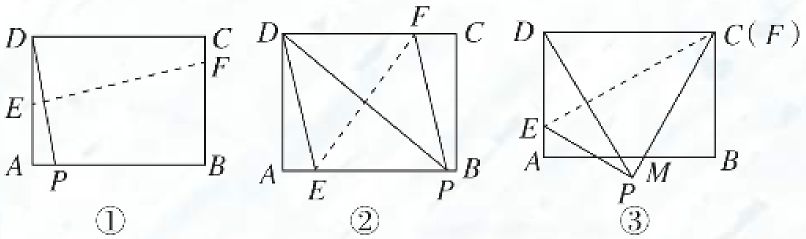
$\therefore$ ▱$DEPF$为菱形.

当$AP=\frac{7}{2}$时，菱形$DEPF$的边长为$\frac{85}{28}$.



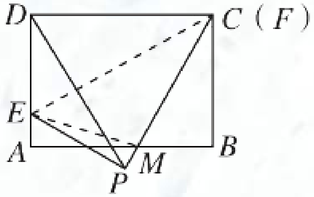
【拓展延伸】

（3）若点F与点C重合，点E在AD上，连结FP，EP，射线BA与射线FP交于点M（如图③）.在折叠过程中，是否存在使得线段AM与线段DE的长度相等的情况？若存在，请求出线段AE的长度；若不存在，请说明理由.



【解】存在.情况一：如图①，

连结EM.



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$\because$ 由题意知$DE=EP=AM$，

$\therefore$ 易得$\triangle EAM\cong\triangle MPE$.

$\therefore AE=PM$.设$AE=PM=x$，则

$AM=DE=AD-AE=3-x$，$\therefore BM=AB-AM=x+1$. $\because$

易知$CP=CD=4$，$\therefore MC=CP-MP=4-x$，在$\mathrm{Rt}\triangle BCM$

中，易得$(x+1)^2+3^2=(4-x)^2$，解得$x=\frac{3}{5}$，即$AE=\frac{3}{5}$；