CHAPTER 17: DYNAMIC ECONOMETRIC MODELS: AUTOREGRESSIVE AND DISTRIBUTED-LAG MODELS

17.1 (*a*) *False*. Econometric models are dynamic if they portray the time path of the dependent variable in relation to its past values. Models using cross-sectional data are not dynamic, unless one uses panel regression models with lagged values of the regressand.

(*b*) *True*. The Koyck model assumes that all the distributed lag coefficients have the same sign.

(*c*) *False*. The estimators are biased as well as inconsistent.

(*d*) *True*. For proof, see the Johnston text cited in footnote # 30.

(*e*) *False*. The method produces consistent estimates, although in small samples the estimates thus obtained are biased.

(*f*) *True*. In such situations, use the Durbin *h* statistic. However, the Durbin *d* statistic can be used in the computation of the *h* statistic.

(*g*) *False*. Strictly speaking, it is valid in large samples.

(*h*) *True*. The Granger test is a measure of precedence and information content but does not, by itself, indicate causality in the common use of the term.

17.2 Make use of Equations (17.7.1), (17.6.2), and (17.5.2).

$$Y_t^* = \beta_0 + \beta_1 X_t^* + u_t \qquad (1)$$

$$Y_t - Y_{t-1} = \delta(Y_t^* - Y_{t-1}) \qquad (2)$$

$$X_t^* - X_{t-1}^* = \gamma(X_t - X_{t-1}^*) \qquad (3)$$

Rewrite Equation (2) as

$$Y_t = \delta Y_t^* + (1-\delta)Y_{t-1} \qquad (4)$$

Rewrite Equation (3) as

$$X_t^* = \frac{\gamma}{1-(1-\gamma)L} X_t \qquad (5)$$

where L is the lag operator such that $LX_t = X_{t-1}$.

Substitute Eq. (1) into Eq. (4) to obtain

$$Y_t = \delta\beta_0 + \delta\beta_1 X_t^* + \delta u_t + (1-\delta)Y_{t-1} \quad (6)$$

Substitute Eq. (5) into Eq. (6) to obtain

$$Y_t = \delta\beta_0 + \delta\beta_1[\frac{\gamma}{1-(1-\gamma)L}X_t] + (1-\delta)Y_{t-1} + \delta u_t \qquad (7)$$

Simplifying Eq. (7), we obtain

$$Y_t = \alpha_1 + \alpha_2 X_t + \alpha_3 Y_{t-1} + \alpha_4 Y_{t-2} \rightarrow (17.7.2)$$

where the α's are (nonlinear) combinations of the various parameters entering into Eq. (7).

17.3 $\text{cov}[Y_{t-1}, (u_t - \lambda u_{t-1})] = E\{[(Y_{t-1} - E(Y_{t-1})][u_t - \lambda u_{t-1}]\}$, since $E(u_t) = 0$.

$= E[(u_{t-1})(u_t - \lambda u_{t-1})]$, since $[Y_{t-1} - E(Y_{t-1})] = u_{t-1}$.

$= -\lambda E[(u_{t-1})^2]$, since there is no serial correlation.

$= -\lambda\sigma^2$.

17.4 The P^* values are 100, 105, 115, 135, and 160, respectively.

17.5 (*a*) The estimated *Y* values, which are a linear function of the the nonstochastic *X* variables, are asymptotically uncorrelated with the population error term, *v*.

(*b*) The problem of collinearity may be less serious.

17.6 (*a*) The median lag is the value of time for which the fraction of adjustment completed is ½. To find the median lag for the Koyck scheme, solve

$$\frac{t \text{ period response}}{\text{long run response}} = \frac{\beta_0(1-\lambda^t)/(1-\lambda)}{\beta_0/(1-\lambda)} = \frac{1}{2}$$

Simplifying, we get

$\lambda^t = \frac{1}{2}$. Therefore,

$t \ln\lambda = \ln(\frac{1}{2}) = -\ln 2$. Therefore,

$t = \frac{-2\ln 2}{\ln\lambda}$, which is the required answer.

(*b*)

λ	$\ln\lambda$	$\ln 2$	Median lag
0.2	-1.6094	0.6932	0.4307
0.4	-0.9163	0.6932	0.7565
0.6	-0.5108	0.6932	1.3569
0.8	-0.2231	0.6932	3.1063

17.7 *(a)* Since $\beta_k = \beta_0 \lambda^k; 0 < \lambda < 1; k = 0,1,2...$

$$\text{mean lag} = \frac{\sum_0^{\infty} k\beta_k}{\sum_0^{k} \beta_k} = \frac{\beta_0 \sum k\lambda^k}{\beta_0 \sum \lambda^k} = \frac{\lambda/(1-\lambda^2)}{1/(1-\lambda)} = \frac{\lambda}{1-\lambda}$$

(b) If λ is very large, the speed of adjustment will be slow.

17.8 Use the formula $\dfrac{\sum k\beta_k}{\sum \beta_k}$. For the data of Table 17.1, this becomes:

$$\frac{11.316}{1.03} = 10.986 \approx 10.959$$

17.9 (*a*) Following the steps in Exercise 17.2, we can write the equation for M_t as:

$$M_t = \alpha + \frac{\beta_1(1-\gamma_1)}{1-\gamma_1 L} Y_t + \frac{\beta_2(1-\gamma_2)}{1-\gamma_2 L} R_t + u_t$$

which can be written as:

$$\begin{aligned} M_t &= \beta_0 + \beta_1(1-\gamma_1)Y_t - \beta_1\gamma_2(1-\gamma_1)Y_{t-1} + \beta_2(1-\gamma_2)R_t \\ &\quad - \beta_2\gamma_1(1-\gamma_2)R_{t-1} + (\gamma_1+\gamma_2)M_{t-1} - (\gamma_1\gamma_2)M_{t-2} + \\ &\quad + [u_t - (\gamma_1+\gamma_2)u_{t-1} + (\gamma_1\gamma_2)u_{t-2}] \end{aligned}$$

where β_0 is a combination of α, γ_1, and γ_2.

Note that if $\gamma_1 = \gamma_2 = \gamma$, the model can be further simplified.

(*b*) The model just developed is highly nonlinear in the parameters and needs to be estimated using some nonlinear iterative procedure as discussed in Chapter 14.

17.10 The estimation of Eq. (17.7.2) poses the same estimation problem as the Koyck or adaptive expectations model in that each is autoregressive with similar error structure. The model is intrinsically a nonlinear regression model, requiring nonlinear estimation techniques.

17.11 As explained by Griliches, since the serial correlation model includes lagged values of the regressors and the Koyck and partial adjustment models do not, the serial correlation model may be appropriate in situations where we are transforming a model to get rid of (first-order) serial correlation, even though it may resemble the Koyck or the PAM.

17.12 (*a*) Yes, in this case the Koyck model may be estimated with OLS.

(*b*) There will be a finite sample bias due to the lagged regressand, but the estimates are consistent. The proof can be found in Henri Theil, Principles *of Econometrics*, John Wiley & Sons, New York, 1971, pp. 408-411.

(*c*)Since both ρ and λ are assumed to lie between 0 and 1, the assumption that they both are equal is plausible.

17.13 Similar to Koyck, Alt, Tinbergen, and other models, this approach is ad hoc and has little theoretical underpinning. It assumes that the importance of the past values declines continuously from the beginning, which may a reasonable assumption is some cases. By using the weighted average of current and past explanatory variables, this triangular model avoids the problems of multicollinearity that may be present in other models.

17.14 (*a*) On average, over the sample period, the change in employment is positively related to output, negatively related to employment in the previous period and negatively related to time . The negative sign of the time coefficient and the negative sign of the time-squared variable suggest that over the sample period the change in employment has been declining, but declining at a faster rate. Note that the time coefficient is not significant at the 5% level, but the time-squared coefficient is.

(*b*) It is 0.297

(*c*)To obtain the long-run demand curve, divide the short-run demand function through by δ and drop the lagged employment term. This gives the long-run demand function as:

$$47.879 + 0.579Q_t + 0.094t + 0.002t^2$$

(*d*) The appropriate test statistic here is the Durbin *h.* Given that $n = 44$ and $d = 1.37$, we obtain:

$$h = (1 - \frac{d}{2})\sqrt{\frac{n}{1 - n\,\mathrm{var}(coeff\ of\ E_{t-1})}}$$

$$= [1\text{-}\frac{1}{2}d\,]\sqrt{\frac{44}{1 - 44(0.001089)}} = 2.414$$

Since h asymptotically follows the normal distribution, the 5% critical value is 1.96. Assuming the sample of 44 observations is reasonably large, we can conclude that there is evidence of first-order positive autocorrelation in the data.