Lecture #4 Summary

$$PM(t) = cos\left(\omega_c t + k_p S(t)\right)$$

$$FM(t) = cos\left(\omega_c t + k_f \int dt\ S(t)\right)$$ with $$\omega(t) = \frac{d\theta}{dt} = \omega_c + k_f S(t)$$

For small modulation indices, the bandwidth is the same as DSB-AM

$$\beta_{FM} = \frac{\Delta_f}{f_m} \qquad \beta_{PM} = k_p$$

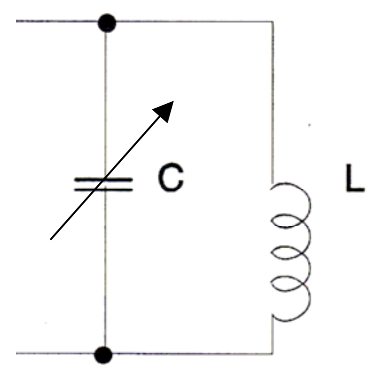
Wideband case: sideband amplitudes given by $J_n(\beta)$

Direct generation of FM

We must now turn our attention to how we actually build circuits to do what we want, in this case to modulate and demodulate FM and PM.

An intuitive approach to generating FM is to build an oscillator whose frequency is changeable through a control voltage, i.e. a *voltage controlled oscillator, VCO.*

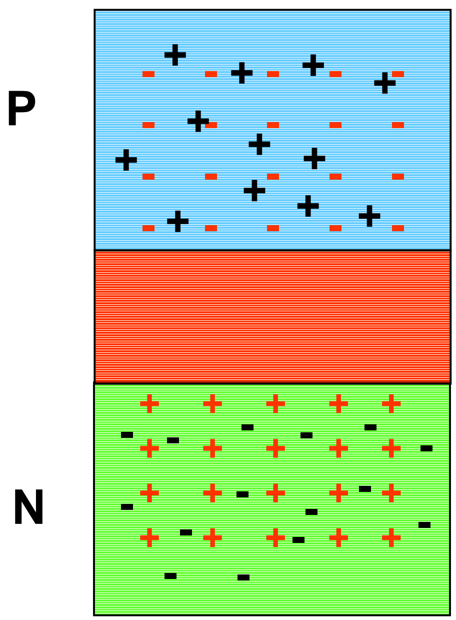
Many oscillators have their frequency set by a parallel LC combination so a VCO actually requires a voltage controlled L or C



It is difficult to get a voltage dependent L but we can get voltage controlled C

Voltage controlled capacitance?

Consider a PN junction diode

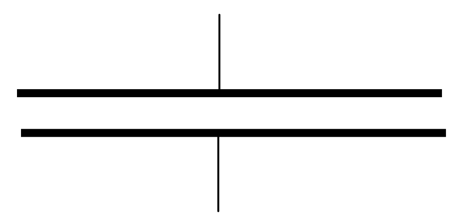


Lots of holes that can move – good conductor

No *moveable* holes or electrons - insulator

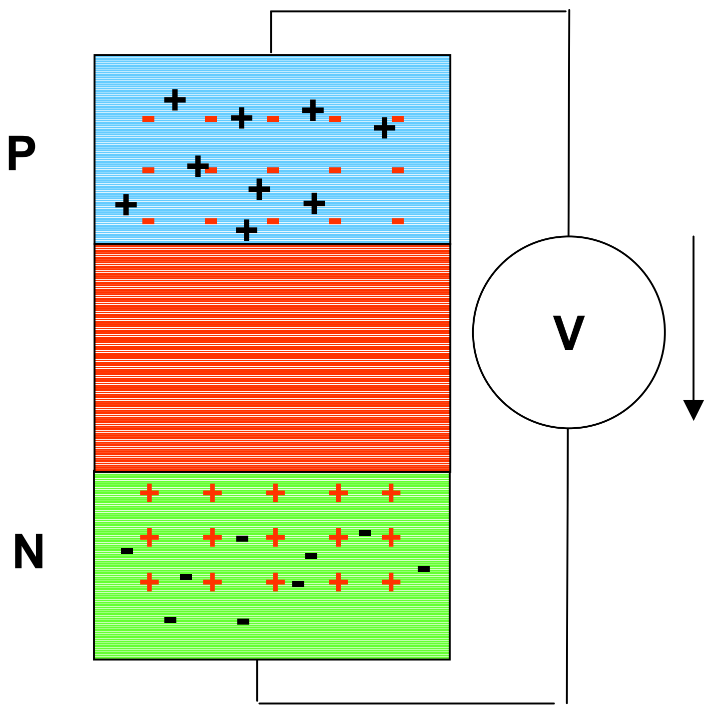
Lots of electrons that can move – good conductor

$$C \propto \frac{Area}{separation}$$



Voltage controlled capacitance?

Add reverse bias

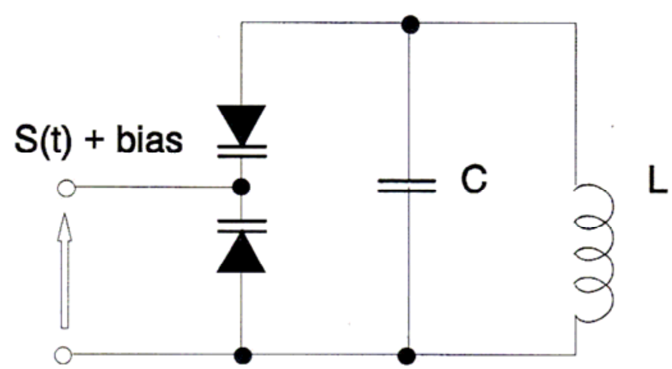


$$C \propto \frac{Area}{separation}$$

Lower capacitance

Angle Modulator Circuits

Below we have a parallel LCR circuit which can be used in an oscillator. The two diodes, called *varicap* or *varactor* diodes are designed such they present a capacitance that changes with the voltage across them



If the modulating signal is applied across them the resonant frequency of the tank circuit and hence that produced by the oscillator will vary with the modulating signal

The resonant frequency of the circuit is given by

$$\omega_c = \frac{1}{\sqrt{LC}}$$

where C is the combination of C_o and the diode capacitance. If C_o is the capacitance when no modulating signal is applied, then with it applied $C = C_o + \Delta C = C_o + K\,S(t)$.

Angle Modulator Circuits

C = C_o + ΔC = C_o + K S(t).

$$\omega_c = \frac{1}{\sqrt{LC}}$$

Therefore

$$\omega_c = \frac{1}{\sqrt{LC_o}\sqrt{1+\frac{\Delta C}{C_o}}}$$

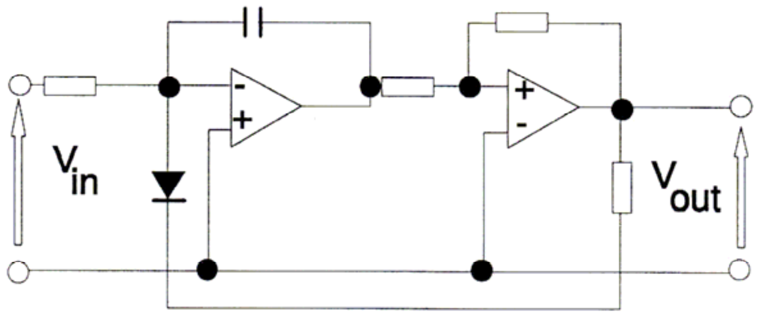
Usually ΔC << C_o such that a binomial expansion can be made

$$\omega_c \approx \frac{1}{\sqrt{LC_o}}\left(1 - \frac{\Delta C}{2C_o}\right) = \omega_o\left(1 - \frac{K}{2C_o}S(t)\right)$$

Therefore the resonant frequency is directly proportional to S(t)

Angle Modulator Circuits

This approach is limited to producing NBFM as the practical range of ΔC is small. At lower frequencies, circuits of the form shown below are also possible for direct FM generation.



Generally, the difficulties with direct FM generation are associated with frequency stability and harmonic purity.

Indirect generation

An interesting alternative to the direct generation of FM is *indirect* generation of FM.

The idea is to construct narrowband FM, which we found consists of three frequency components, by generating these components separately and then adding them together.

Recall that narrow band FM is given by

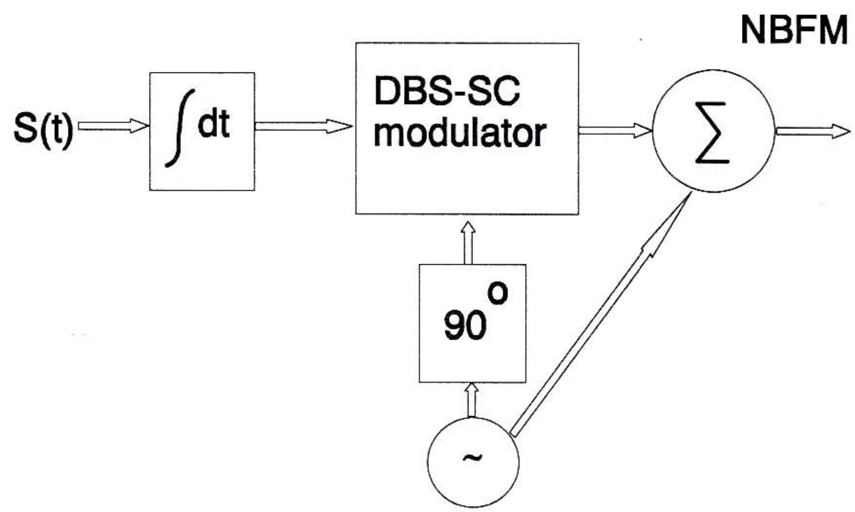
$$NBFM(t) \approx \cos(\omega_c t) + \sin(\omega_c t)\, k_f \int dt\, S(t))$$

i.e. it is the sum of a carrier term and the product of a quadrature carrier with the modulating signal.

Indirect generation

$$NBFM(t) \approx \cos(\omega_c t) + \sin(\omega_c t)\, k_f \int dt\, S(t))$$

The multiply we know can be done using a DSB+SC modulator and so we can use the circuit given below. This method is also called *Armstrong FM* after one of the gurus of FM in the 30's.



Narrow band PM can also be generated in the same way by just taking out the integrator.

Wideband Modulation

Unfortunately, we cannot generate the more useful wideband FM in this way.

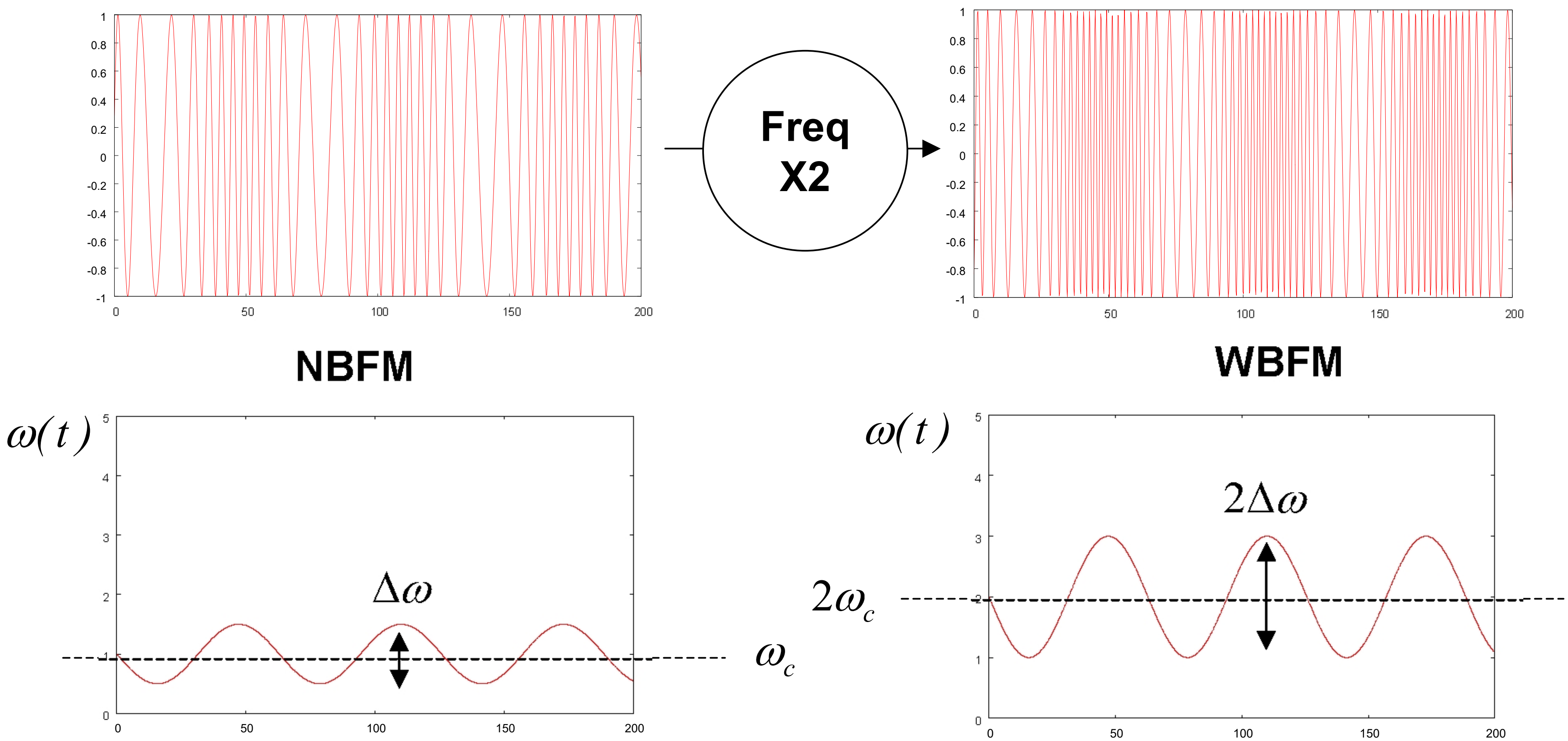
However, it is possible to build circuits, *frequency multipliers,* which will double, treble etc the *instantaneous* frequency of a signal which we can exploit.

$\beta = \frac{\Delta\omega}{\omega_m}$ can be increased if $\Delta\omega$ can be increased

Therefore, if we follow a narrow band FM generator with a times n frequency multiplier, we can produce wider band FM

Wideband Modulation

For example, if the peak frequency deviation of the NBFM is $\pm\,\Delta\omega$ when the modulating signal is cos $(\omega_m t)$, then the modulation index is $\Delta\omega/\omega_m$.

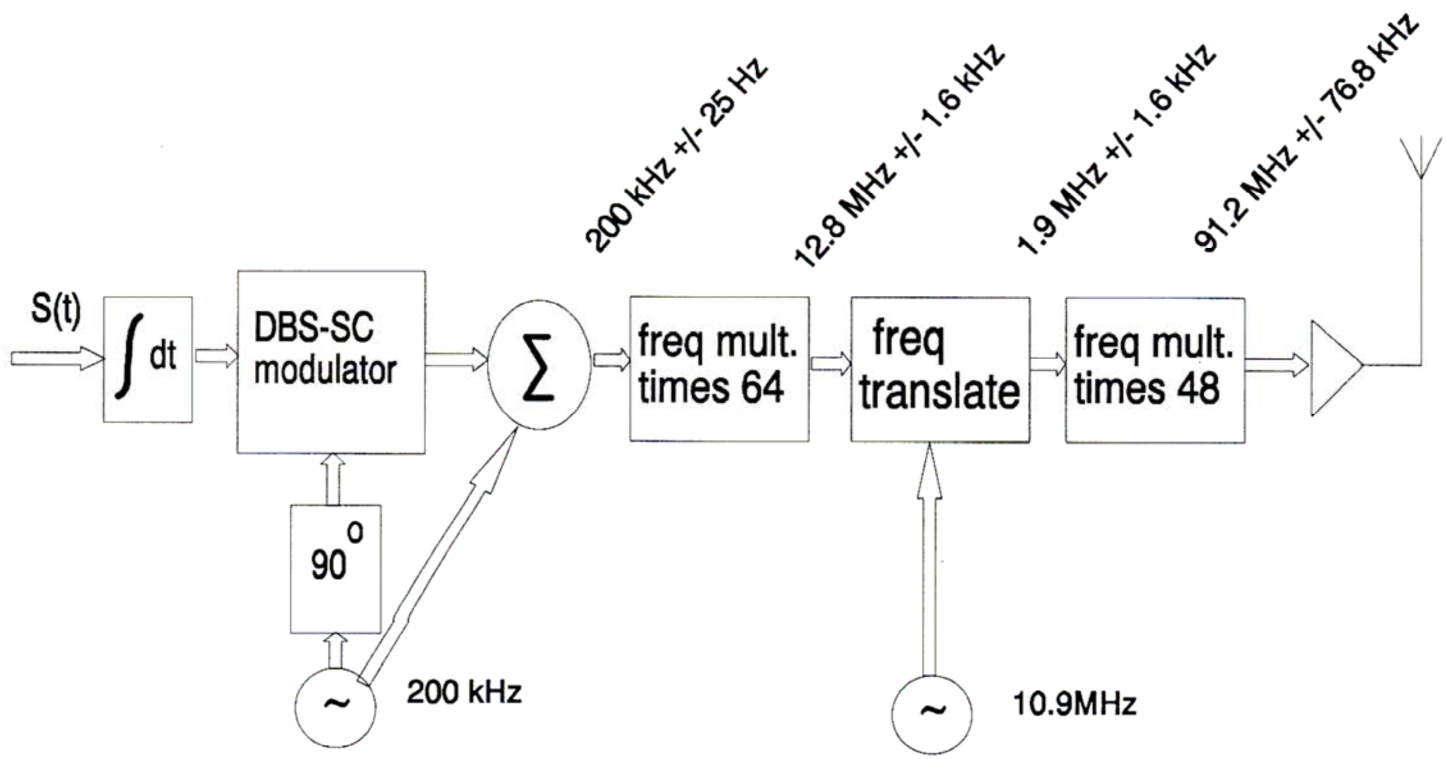


$$\beta = \frac{\Delta\omega}{\omega_m} \qquad\qquad \beta = \frac{2\Delta\omega}{\omega_m}$$

Wideband Modulation

We note that after the first frequency multiplier, not only is the modulation index suitably increased, but obviously so is the centre frequency.

This can be compensated for by a *frequency translator*, which just translates the whole frequency spectrum up or down to the desired final value A practical example of a FM transmitter using this idea is



FM demodulation

How to reverse the process?

The easiest way is to first of all convert the incoming FM signal to an AM and then use an AM demodulator.

To convert FM to AM we need a circuit whose output voltage is proportional to the frequency of the constant amplitude input voltage.

We can actually use part of the frequency response of an LCR filter as shown