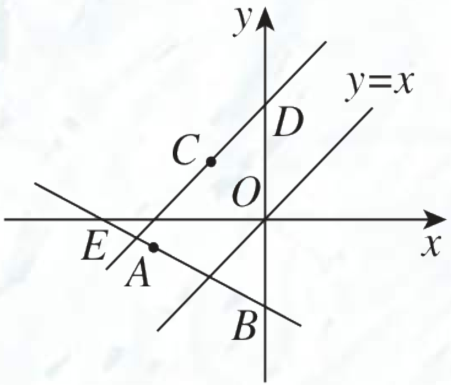
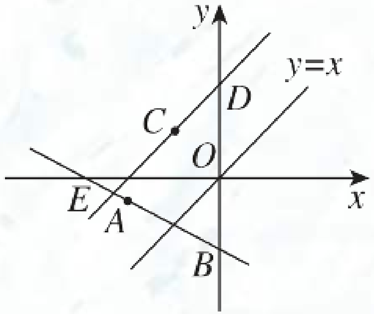
阶段拔尖专训5 一次函数与几何大综合

题型1 探究点的坐标

1.如图，在平面直角坐标系中，直线$y=-\frac{1}{2}x-3$过点$A(-4,m)$且与y轴交于点B，把点A向右平移2个单位长度，再向上平移3个单位长度，得到点C，过点C且与直线$y=x$平行的直线交y轴于点D.



（1）求直线CD的表达式；



【解】把$A(-4,m)$的坐标代入$y=-\frac{1}{2}x-3$，

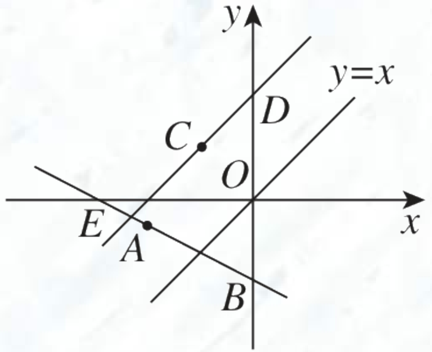
得$m=2-3=-1$，$\therefore A(-4,-1)$. $\because$ 点A向右平移2个单位长度，再向上平移3个单位长度，得到点C，

$\therefore C(-2,2)$. $\because$ 过点C且与直线$y=x$平行的直线交y轴于点D，

$\therefore$ 设直线CD的表达式为$y=x+b$.把$C(-2,2)$的坐标代入，得

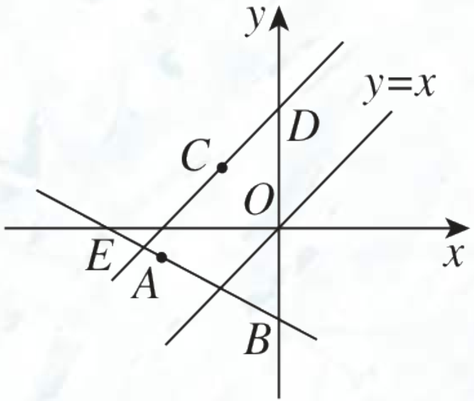
$-2+b=2$，解得$b=4$，$\therefore$ 直线CD的表达式为$y=x+4$.

(2) 直线AB与CD交于点E，将直线CD沿射线EB方向平移，平移到经过点B的位置结束，求直线CD在平移过程中与x轴交点的横坐标的取值范围.



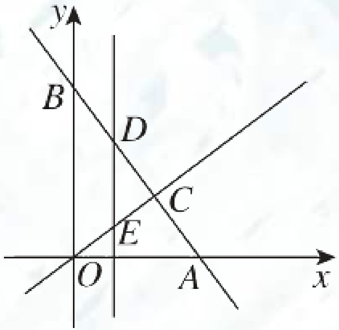
对于$y=-\frac{1}{2}x-3$，当$x=0$时，$y=-3$，

$\therefore B(0,-3)$.



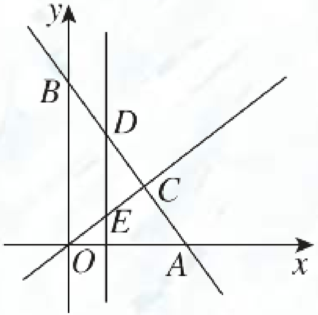
对于$y=x+4$，当$y=0$时，$x=-4$，$\therefore$ 直线CD与x轴交点的横坐标为-4.易得直线CD沿射线EB方向平移到经过点B时的直线表达式为$y=x-3$. 当$y=0$时，$x=3$，$\therefore$ 直线$y=x-3$与x轴交点的横坐标为3，$\therefore$ 直线CD在平移过程中与x轴交点的横坐标的取值范围为$-4\leq x\leq 3$.

2.[2024成都期末] 如图，直线$y = kx + b$经过点$B(0,25)$，与直线$y = \frac{3}{4}x$交于点$C(m, 9)$，与x轴交于点A，点D为直线AB上一动点，过点D作x轴的垂线交直线OC于点E.



（1）求点A的坐标；

【解】$\because$ 直线$y=\frac{3}{4}x$过点$C(m,9)$，

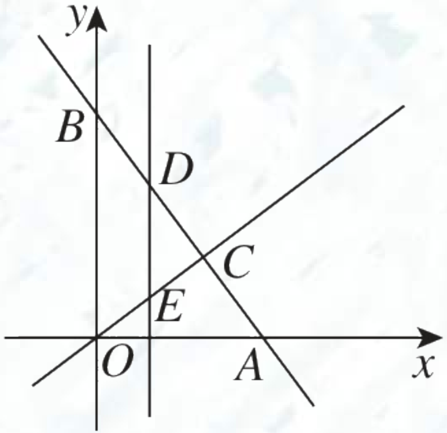


$\therefore 9=\frac{3}{4}m$，解得$m=12.\therefore C(12,9)$.

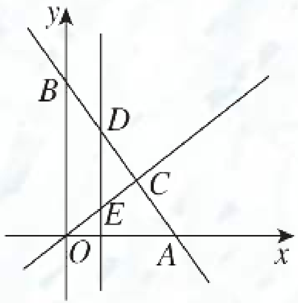
$\because$ 直线$y=kx+b$经过点$B(0,25)$，$C(12,9)$，

$\therefore \begin{cases}12k+b=9,\\ b=25,\end{cases}$解得$\begin{cases}k=-\frac{4}{3},\\ b=25.\end{cases}$ $\therefore$ 直线AB的表达式为$y=-\frac{4}{3}x+25$.令$y=0$，则$x=\frac{75}{4}$，$\therefore$ 点A的坐标为$(\frac{75}{4},0)$.

(2) 当$DE=\frac{1}{2}OB$时，求△CDE的面积；



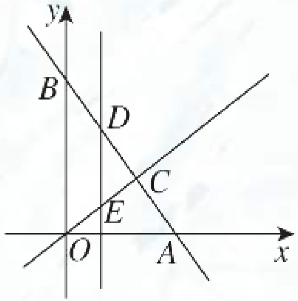
$\because B(0,25)$，$\therefore OB=25.\because DE=\frac{1}{2}OB$，



$\therefore DE=\frac{25}{2}$.

设$D(n,-\frac{4}{3}n+25)$，则$E(n,\frac{3}{4}n)$，

$\therefore DE=|-\frac{4}{3}n+25-\frac{3}{4}n|=|-\frac{25}{12}n+25|=\frac{25}{2}$



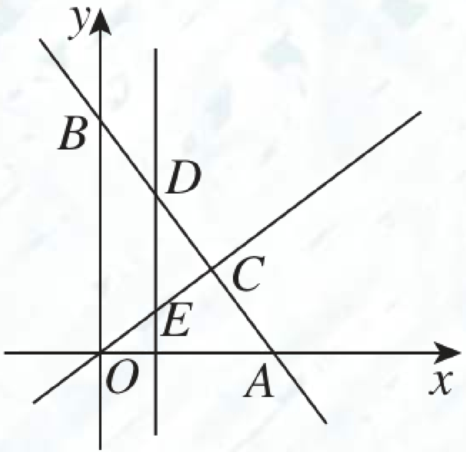
解得$n=6$或18.当$n=6$时，

$S_{\triangle CDE}=\frac{1}{2}\times\frac{25}{2}\times(12-6)=\frac{75}{2}$；当$n=18$

时，$S_{\triangle CDE}=\frac{1}{2}\times\frac{25}{2}\times(18-12)=\frac{75}{2}$.

综上，$\triangle CDE$的面积为$\frac{75}{2}$.

（3）连结OD，若将$\triangle OAD$沿着OD折叠，使得点A的对应点A_1落在直线OC上，求点D的坐标.



过点C作$CG \perp OA$于点G. $\because C(12,9)$，$A(\frac{75}{4},0)$，$\therefore OG = 12$，

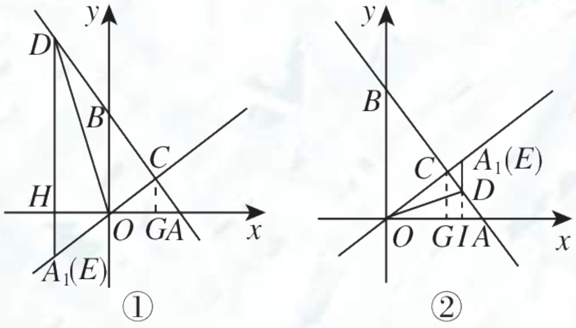
$CG = 9$，$OA = \frac{75}{4}$，$\therefore AG = \frac{75}{4} - 12 = \frac{27}{4}$.

易知$OC^2 = OG^2 + CG^2 = 225$，$AC^2 = AG^2 + CG^2 = \frac{2\,025}{16}$，

$OA^2 = \frac{5\,625}{16}$. $\therefore OC = 15$，$OC^2 + AC^2 = \frac{5\,625}{16} = OA^2$.

$\therefore \angle OCA = 90°$，即$OC \perp AB$.

易知点A_1不可能在原点处，当点A_1在第三象限时，设DA_1交x轴于点H，如图①所示.



根据折叠的性质可得$OA = OA_1$，$\angle DAO = \angle DA_1O$.

又$\because \angle COA = \angle HOA_1$，$\therefore \triangle COA \cong \triangle HOA_1$.

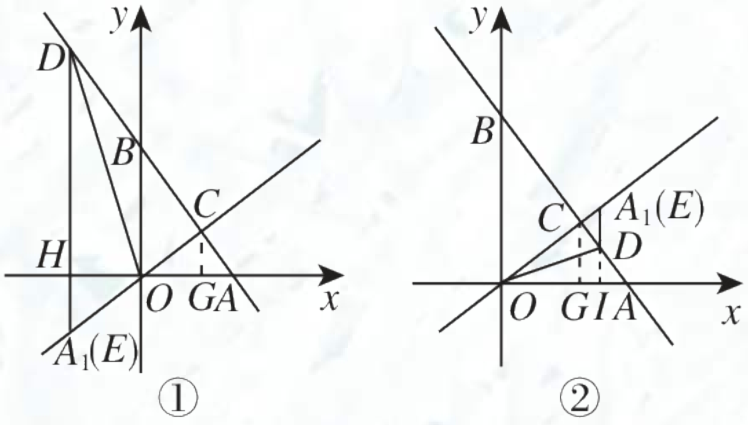
$\therefore \angle A_1HO = \angle ACO = 90^\circ$，$HO = CO = 15$. $\therefore DA_1 \perp x$轴.对

于$y=-\frac{4}{3}x+25$，当$x=-15$时，$y=45$，∴ 点D的坐标为$(-15,45)$；

当点A_1在第一象限时，延长A_1D交x轴于点I，如图②所示.

根据折叠的性质可得

$OA=OA_1$，$\angle DAO=\angle DA_1O$.



又$\because \angle COA=\angle IOA_1$，

$\therefore \triangle COA \cong \triangle IOA_1$.

$\therefore \angle A_1IO=\angle ACO=90°$，

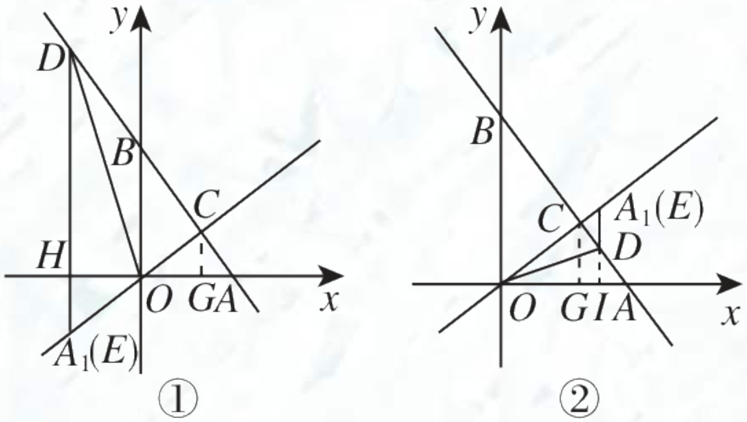
$IO=CO=15$. $\therefore A_1I \perp x$轴.对于

$y=-\frac{4}{3}x+25$，当$x=15$时，

$y=5$，$\therefore$ 点D的坐标为$(15,5)$.

综上，点D的坐标为$(-15,45)$或

$(15,5)$.



题型2 求表达式中的待定系数的值

3.在平面直角坐标系xOy中，函数$y = kx + b(k \neq 0)$的图象经过点$A(0,1)$和$B(1,2)$，与过点$(0,4)$且平行于x轴的直线交于点C.

（1）求该函数的表达式及点C的坐标;

【解】把点$A(0,1)$,$B(1,2)$的坐标分别代入$y=kx+b(k\neq 0)$,

得$\begin{cases} b=1, \\ k+b=2, \end{cases}$解得$\begin{cases} k=1, \\ b=1, \end{cases}$

$\therefore$ 该函数的表达式为$y=x+1$.

由题意知,点C的纵坐标为4,

当$y=4$时,$x=3$. $\therefore C(3,4)$.

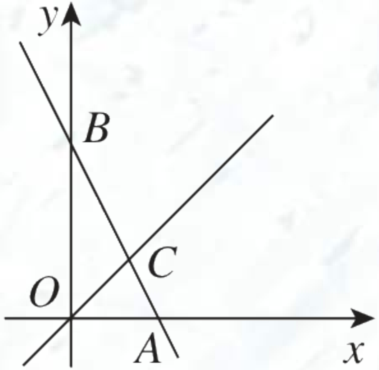
（2）当$x < 3$时，对于x的每一个值，函数$y = \frac{2}{3}x + n$的值大于函数$y = kx + b(k \neq 0)$的值且小于4,求n的值.

由题意得，直线$y = \frac{2}{3}x + n$过点$(3,4)$，

$\therefore 4 = \frac{2}{3} \times 3 + n$,解得$n = 2$.

题型3 动点最值问题

4.在平面直角坐标系中，直线AB与x轴交于点A，与y轴交于点B，与直线OC交于点C.



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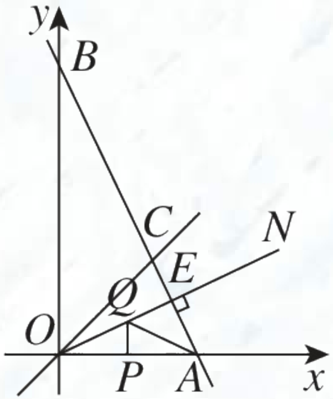

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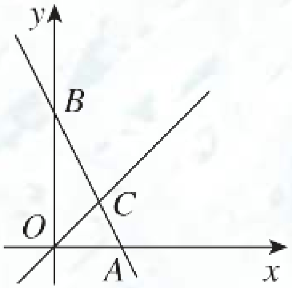
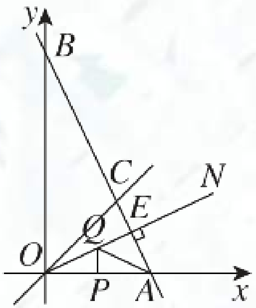
（1）如图①，若直线AB的表达式为$y=-2x+12$，直线OC的表达式为$y=x$.

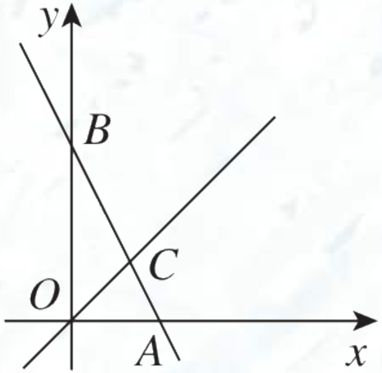




①点C的坐标为__(4,4)__；

②$\triangle OAC$的面积=__12__；

(2) 如图②，作$\angle AOC$的平分线ON，若$AB \perp ON$，垂足为E，$OA = 4$，$\triangle OAC$的面积为6，P，Q分别为线段OA，OE上的动点，连结AQ，PQ，求$AQ + PQ$的最小值.



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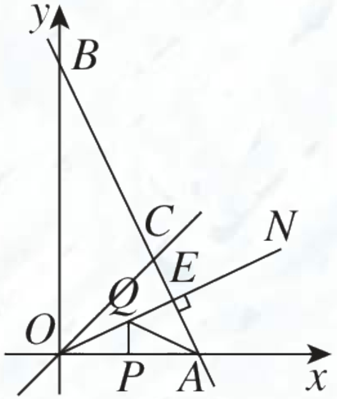

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【解】连结CQ.

$\because ON$平分$\angle AOC, ON \perp AB$，$\therefore$ 易得ON垂直平分AC.

$\therefore CQ = AQ$.

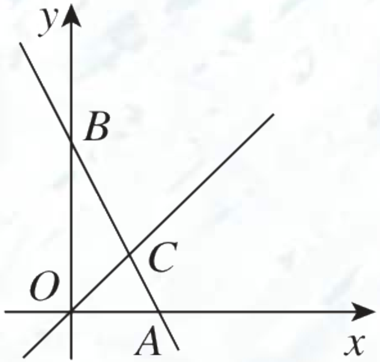
$\therefore AQ + PQ = CQ + PQ$.

当C, Q, P在同一直线上，且$CP \perp OA$时，$CQ + PQ$的值最小.

$\because \triangle OAC$的面积为6，$OA = 4$，$CP \perp OA$，

$\therefore CP = 3. \therefore CQ + PQ$的最小值为3.

$\therefore AQ + PQ$的最小值为3.



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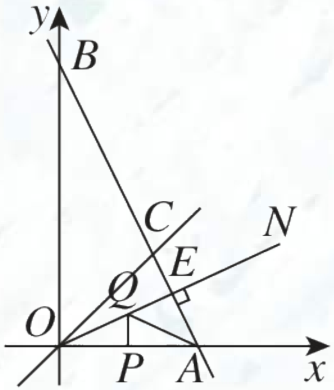

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