

三角形中的倒角模型 — 平行线 + 拐点模型

近年来各地中考中常出现一些几何倒角模型,该模型主要涉及高线、角平分线及角度的计算(内角和定理、外角定理等)。平行线+拐点模型在初中数学几何模块中属于基础工具类问题,也是学生必须掌握的一块内容,熟悉这些模型可以快速得到角的关系,求出所需的角。本专题就平行线+拐点模型(猪蹄模型(M型)、铅笔头模型、牛角模型、羊角模型、“5”字模型)进行梳理及对应试题分析,方便掌握。

拐点(平行线)模型的核心是一组平行线与一个点,然后把点与两条线分别连起来,就构成了拐点模型,这个点叫做拐点,两条线的夹角叫做拐角。

通用解法:见拐点作平行线;**基本思路:**和差拆分与等角转化。

模型1:猪蹄模型(M型)

【模型解读】

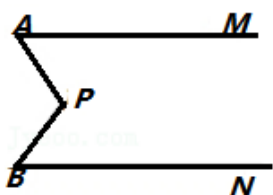


图1

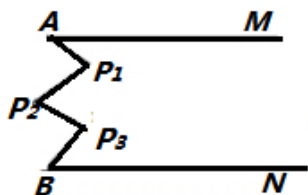


图2

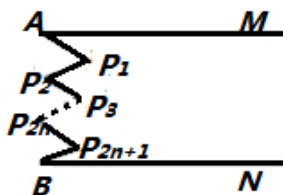


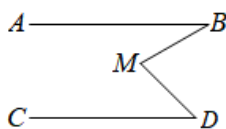
图3

如图1,①已知: $AM \parallel BN$, 结论: $\angle APB = \angle A + \angle B$; ②已知: $\angle APB = \angle A + \angle B$, 结论: $AM \parallel BN$.

如图2, 已知: $AM \parallel BN$, 结论: $\angle P_1 + \angle P_3 = \angle A + \angle B + \angle P_2$.

如图3, 已知: $AM \parallel BN$, 结论: $\angle P_1 + \angle P_3 + \dots + \angle P_{2n+1} = \angle A + \angle B + \angle P_2 + \dots + \angle P_{2n}$.

例1 (2022·河南洛阳·统考二模) 如图, $AB \parallel CD$, $\angle ABM = 30^\circ$, $\angle CDM = 45^\circ$, 则 $\angle BMD$ 的度数为 ()



A. 105°

B. 90°

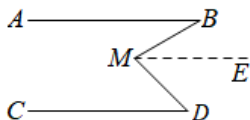
C. 75°

D. 70°

【答案】C

【分析】过点M作 $ME \parallel AB$, 从而可得 $AB \parallel ME \parallel CD$, 则有 $\angle ABM = \angle BME$, $\angle CDM = \angle DME$, 即可求 $\angle BMD$ 的度数.

【详解】解: 过点M作 $ME \parallel AB$, 如图,



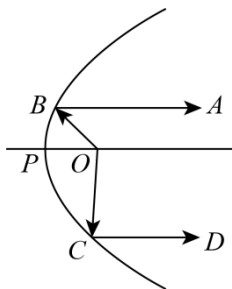
$\because AB \parallel CD, \therefore AB \parallel ME \parallel CD, \therefore \angle ABM = \angle BME = 30^\circ, \angle CDM = \angle DME = 45^\circ,$

$\therefore \angle BMD = \angle BME + \angle DME = 75^\circ$. 故选: C.

【点睛】本题考查平行线的性质,解答的关键是熟记平行线的性质并灵活运用.

例2 (2023春·安徽蚌埠·九年级校联考期中) 太阳灶、卫星信号接收锅、探照灯及其他很多灯具都与抛物线有关. 如图, 从点O照射到抛物线上的光线OB, OC反射后沿着与PO平行的方向射出, 已知图中 $\angle ABO$

$= 46^\circ$, $\angle OCD = 88^\circ$, 则 $\angle BOC$ 的度数为 ()



- A. 116° B. 124° C. 134° D. 135°

【答案】C

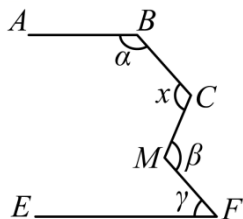
【分析】由平行线的性质即可得出 $\angle BOP = 46^\circ$, $\angle COP = 88^\circ$, 再根据 $\angle BOC = \angle BOP + \angle COP$ 即可求解.

【详解】由题意知 $AB \parallel PO \parallel CD \therefore \angle BOP = \angle ABO = 46^\circ$, $\angle COP = \angle OCD = 88^\circ$

$\therefore \angle BOC = \angle BOP + \angle COP = 134^\circ$ 故选: C.

【点睛】题考查了平行线的性质, 两直线平行, 内错角相等, 牢记性质是解决问题的关键.

例 3 (2023 春·四川泸州·七年级校考期末) 如图所示, 若 $AB \parallel EF$, 用含 α 、 β 、 γ 的式子表示 x , 应为 ()



- A. $\alpha + \beta + \gamma$ B. $\beta + \gamma - \alpha$ C. $180^\circ - \alpha - \gamma + \beta$ D. $180^\circ + \alpha + \beta - \gamma$

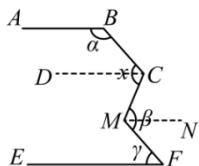
【答案】C

【分析】过 C 作 $CD \parallel AB$, 过 M 作 $MN \parallel EF$, 推出 $AB \parallel CD \parallel MN \parallel EF$, 根据平行线的性质得出 $\alpha + \angle BCD = 180^\circ$, $\angle DCM = \angle CMN$, $\angle NMF = \gamma$, 求出 $\angle BCD = 180^\circ - \alpha$, $\angle DCM = \angle CMN = \beta - \gamma$, 即可得出答案.

【详解】过 C 作 $CD \parallel AB$, 过 M 作 $MN \parallel EF$,

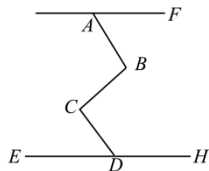
$\because AB \parallel EF, \therefore AB \parallel CD \parallel MN \parallel EF, \therefore \alpha + \angle BCD = 180^\circ, \angle DCM = \angle CMN, \angle NMF = \gamma,$

$\therefore \angle BCD = 180^\circ - \alpha, \angle DCM = \angle CMN = \beta - \gamma, \therefore x = \angle BCD + \angle DCM = 180^\circ - \alpha - \gamma + \beta$, 故选: C.



【点睛】本题考查了平行线的性质的应用, 主要考查了学生的推理能力.

例 4 (2023·广东深圳·校联考模拟预测) 北京冬奥会掀起了滑雪的热潮, 谷爱凌的励志故事也激励着我们青少年, 很多同学纷纷来到滑雪场, 想亲身感受一下奥运健儿在赛场上风驰电掣的感觉, 但是第一次走进滑雪场的你, 如果不想体验人仰马翻的感觉, 学会正确的滑雪姿势是最重要的, 正确的滑雪姿势是上身挺直略前倾, 与小腿平行, 使脚的根部处于微微受力的状态, 如图所示, $AB \parallel CD$, 当人脚与地面的夹角 $\angle CDE = 60^\circ$ 时, 求出此时上身 AB 与水平线的夹角 $\angle BAF$ 的度数为 ()



A. 60°

B. 45°

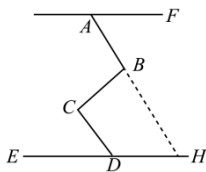
C. 50°

D. 55°

【答案】A

【分析】延长 AB 交直线 ED 于点 H , 利用平行线的性质得出 $\angle CDE = \angle DHA = 60^\circ$, 再由两直线平行, 内错角相等即可得出结果.

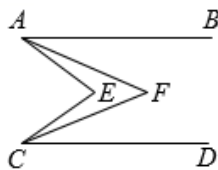
【详解】解: 延长 AB 交直线 ED 于点 H , $\therefore AH \parallel CD$, $\therefore \angle CDE = \angle DHA = 60^\circ$,



\therefore 根据题意得 $AF \parallel EH$, $\therefore \angle FAB = \angle DHA = 60^\circ$, 故选: A.

【点睛】题目考查平行线的性质, 理解题意, 熟练掌握运用平行线的性质是解题关键.

例 5 (2023 春·河南驻马店·九年级专题练习) 已知 $AB \parallel CD$, $\angle EAF = \frac{1}{3}\angle EAB$, $\angle ECF = \frac{1}{3}\angle ECD$, 若 $\angle E = 66^\circ$, 则 $\angle F$ 为 ()



A. 23°

B. 33°

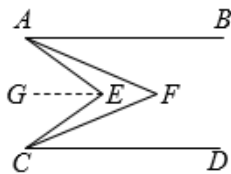
C. 44°

D. 46°

【答案】C

【分析】如图 (见解析), 先根据平行线的性质、角的和差可得 $\angle EAB + \angle ECD = \angle AEC = 66^\circ$, 同样的方法可得 $\angle F = \angle FAB + \angle FCD$, 再根据角的倍分可得 $\angle FAB = \frac{2}{3}\angle EAB$, $\angle FCD = \frac{2}{3}\angle ECD$, 由此即可得出答案.

【详解】如图, 过点 E 作 $EG \parallel AB$, 则 $EG \parallel AB \parallel CD$,



$\therefore \angle AEG = \angle EAB$, $\angle CEG = \angle ECD$, $\therefore \angle EAB + \angle ECD = \angle AEG + \angle CEG = \angle AEC = 66^\circ$,

同理可得: $\angle F = \angle FAB + \angle FCD$, $\therefore \angle EAF = \frac{1}{3}\angle EAB$, $\angle ECF = \frac{1}{3}\angle ECD$,

$\therefore \angle FAB = \frac{2}{3}\angle EAB$, $\angle FCD = \frac{2}{3}\angle ECD$,

$\therefore \angle F = \angle FAB + \angle FCD = \frac{2}{3}\angle EAB + \frac{2}{3}\angle ECD = \frac{2}{3}(\angle EAB + \angle ECD) = \frac{2}{3} \times 66^\circ = 44^\circ$, 故选: C.

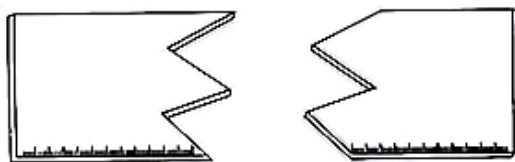
【点睛】本题考查了平行线的性质、角的和差倍分, 熟练掌握平行线的性质是解题关键.

例 6 (2022·浙江七年级期中) 如图(1)所示是一根木尺折断后的情形, 你可能注意过, 木尺折断后的断口一般是参差不齐的, 那么请你深入考虑一下其中所包含的一类数学问题, 我们不妨取名叫“木尺断口问题”.

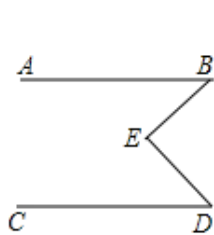
(1) 如图(2)所示, 已知 $AB \parallel CD$, 请问 $\angle B$, $\angle D$, $\angle E$ 有何关系并说明理由;

(2) 如图(3)所示, 已知 $AB \parallel CD$, 请问 $\angle B$, $\angle E$, $\angle D$ 又有何关系并说明理由;

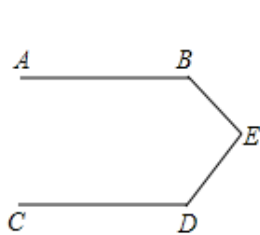
(3) 如图(4)所示, 已知 $AB \parallel CD$, 请问 $\angle E + \angle G$ 与 $\angle B + \angle F + \angle D$ 有何关系并说明理由.



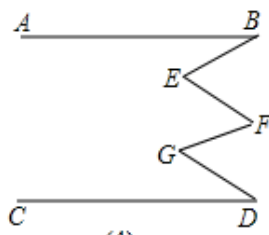
(1)



(2)



(3)

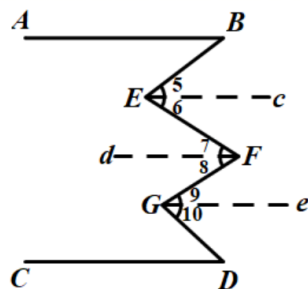
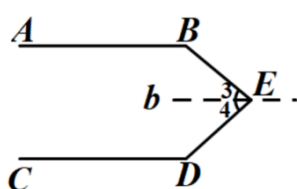
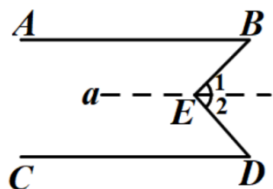


(4)

【答案】见解析.

【解析】解: (1) $\angle E = \angle B + \angle D$, 理由如下:

过点 E 作直线 $a \parallel AB$, 则 $a \parallel AB \parallel CD$, 则 $\angle B = \angle 1$, $\angle D = \angle 2$, $\therefore \angle BED = \angle 1 + \angle 2 = \angle B + \angle D$.



(2) $\angle E + \angle B + \angle D = 360^\circ$, 理由如下: 过点 E 作直线 $b \parallel AB$, 则 $b \parallel AB \parallel CD$. $\therefore \angle B + \angle 3 = 180^\circ$, $\angle 4 + \angle D = 180^\circ$

$\therefore \angle B + \angle 3 + \angle 4 + \angle D = 360^\circ$ 即 $\angle E + \angle B + \angle D = 360^\circ$.

(3) $\angle B + \angle F + \angle D = \angle E + \angle G$, 理由如下:

过点 E, F, G 作直线 $c \parallel AB$, $d \parallel AB$, $e \parallel AB$, 则 $c \parallel AB \parallel d \parallel e \parallel CD$,

则 $\angle B = \angle 5$, $\angle 6 = \angle 7$, $\angle 8 = \angle 9$, $\angle 10 = \angle D$

$\therefore \angle B + \angle EFG + \angle D = \angle 5 + \angle 7 + \angle 8 + \angle 10 = \angle 5 + \angle 6 + \angle 9 + \angle 10 = \angle BEF + \angle FGD$.

模型 2: 铅笔头模型

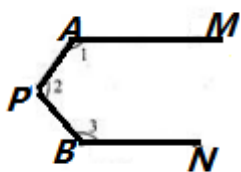


图1

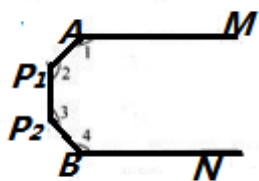


图2

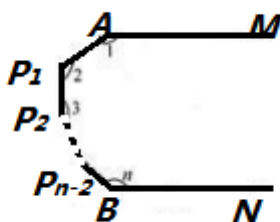


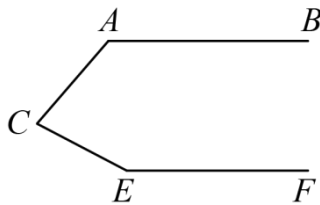
图3

如图1, ①已知: $AM \parallel BN$, 结论: $\angle 1 + \angle 2 + \angle 3 = 360^\circ$; ②已知: $\angle 1 + \angle 2 + \angle 3 = 360^\circ$, 结论: $AM \parallel BN$.

如图2, 已知: $AM \parallel BN$, 结论: $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 540^\circ$

如图3, 已知: $AM \parallel BN$, 结论: $\angle 1 + \angle 2 + \dots + \angle n = (n-1)180^\circ$.

例 7 (2023·广东·统考二模) 如图所示, 已知 $AB \parallel EF$, 那么 $\angle BAC + \angle ACE + \angle CEF = (\quad)$



A. 180°

B. 270°

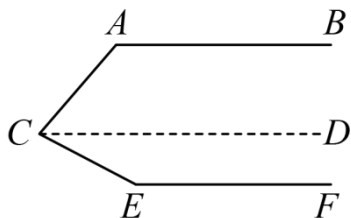
C. 360°

D. 540°

【答案】C

【分析】先根据平行线的性质得出 $\angle BAC + \angle ACD = 180^\circ$, $\angle DCE + \angle CEF = 180^\circ$, 进而可得出结论.

【详解】过点 C 作 $CD \parallel EF$,



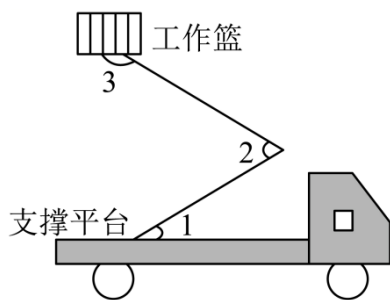
$$\because AB \parallel EF, \therefore AB \parallel CD \parallel EF, \therefore \begin{cases} \angle BAC + \angle ACD = 180^\circ \text{ ①,} \\ \angle DCE + \angle CEF = 180^\circ \text{ ②,} \end{cases}$$

由①+②得, $\angle BAC + \angle ACD + \angle DCE + \angle CEF = 360^\circ$,

即 $\angle BAC + \angle ACE + \angle CEF = 360^\circ$. 故选: C.

【点睛】本题考查的是平行线的性质, 用到的知识点为: 两直线平行, 同旁内角互补.

例 8 (2023·山西吕梁·校联考模拟预测) 如图, 这是路政工程车的工作示意图, 工作篮底部与支撑平台平行. 若 $\angle 1 = 32^\circ$, $\angle 2 = 62^\circ$, 则 $\angle 3$ 的度数为 ()

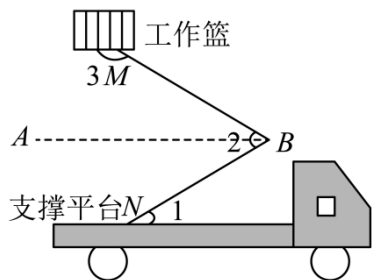


- A. 118° B. 148° C. 150° D. 162°

【答案】C

【分析】过点B作BA//工作篮底部,根据平行线的性质及角的和差求解即可.

【详解】解:如图,过点B作BA//工作篮底部, $\therefore \angle 3 + \angle MBA = 180^\circ$,



\because 工作篮底部与支撑平台平行, $BA \parallel$ 工作篮底部 $\therefore BA \parallel$ 支撑平台, $\therefore \angle ABN = \angle 1 = 32^\circ$,

$\because \angle 2 = \angle ABN + \angle MBA$, $\angle 2 = 62^\circ$, $\therefore \angle MBA = 30^\circ$, $\therefore \angle 3 = 150^\circ$, 故选: C.

【点睛】此题考查了平行线的性质,熟记“两直线平行,内错角相等”、“两直线平行,同旁内角互补”是解题的关键.

例 9 (2023·河南三门峡·校联考·一模) 如图,图1是某小区车库门口的“曲臂直杆道闸”,可抽象为图2所示的数学图形. 已知CD垂直地面上的直线DF于点D,当车牌被自动识别后,曲臂直杆道闸的BC段将绕点C缓慢向上抬高,AB段则一直保持水平状态上升(即AB始终平行于DF). 在该运动过程中,当 $\angle ABC = 112^\circ$ 时, $\angle BCD$ 的度数是 ()



图1

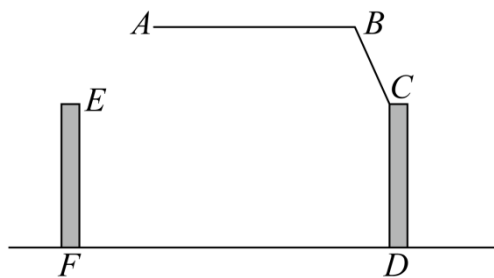


图2

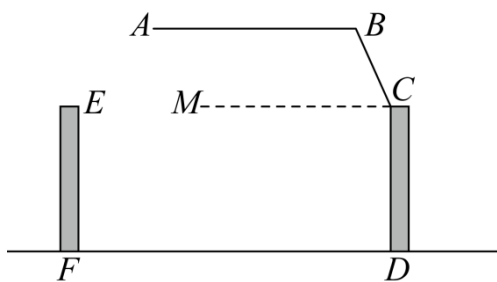
- A. 112° B. 138° C. 158° D. 128°

【答案】C

【分析】如图所示,过点C作 $CM \parallel AB$,利用平行线的性质得到 $\angle ABC + \angle BCM = 180^\circ$, $\angle MCD + \angle CDF = 180^\circ$,进而求出 $\angle BCM = 68^\circ$, $\angle MCD = 90^\circ$,则 $\angle BCD = \angle BCM + \angle MCD = 158^\circ$.

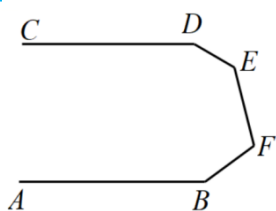
【详解】解:如图所示,过点C作 $CM \parallel AB$, $\because DF \parallel AB$, $\therefore CM \parallel AB \parallel DF$, $\therefore \angle ABC + \angle BCM = 180^\circ$, $\angle MCD + \angle CDF = 180^\circ$,

$\because \angle ABC = 112^\circ$, $CD \perp DF$ 即 $\angle CDF = 90^\circ$, $\therefore \angle BCM = 68^\circ$, $\angle MCD = 90^\circ$,
 $\therefore \angle BCD = \angle BCM + \angle MCD = 158^\circ$, 故选 C.



【点睛】本题主要考查了平行线的性质，熟知两直线平行，同旁内角互补是解题的关键。

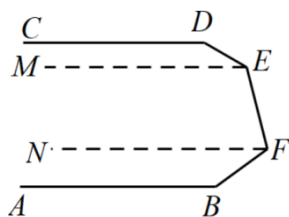
例 10 (2023 春·新疆·七年级校考阶段练习) 如图，如果 $AB \parallel CD$ ，那么 $\angle B + \angle F + \angle E + \angle D =$ _____°.



【答案】540

【分析】过点 E 作 $EM \parallel CD$ ，过点 F 作 $FN \parallel CD$ ，再根据两直线平行，同旁内角互补即可作答。

【详解】过点 E 作 $EM \parallel CD$ ，过点 F 作 $FN \parallel CD$ ，如图，



$\because AB \parallel CD$, $EM \parallel CD$, $FN \parallel CD$, $\therefore AB \parallel FN$, $EM \parallel FN$,

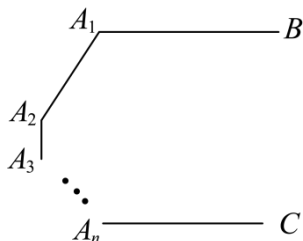
$\therefore \angle B + \angle BFN = 180^\circ$, $\angle FEM + \angle EFN = 180^\circ$, $\angle D + \angle DEM = 180^\circ$,

$\therefore \angle DEF = \angle DEM + \angle FEM$, $\angle BFE = \angle BFN + \angle EFN$,

$\therefore \angle B + \angle BFE + \angle DEF + \angle D = \angle B + \angle BFN + \angle FEM + \angle EFN + \angle D + \angle DEM = 540^\circ$, 故答案为: 540.

【点睛】本题主要考查了平行线的性质，即两直线平行，同旁内角互补。构造辅助线 $EM \parallel CD$, $FN \parallel CD$ 是解答本题的关键。

例 11 (2022 春·河北保定·七年级校考期中) 如图，已知 $A_1B \parallel A_nC$ ，则 $\angle A_1 + \angle A_2 + \angle A_3 =$ _____，则 $\angle A_1 + \angle A_2 + \dots + \angle A_n$ 等于 _____ (用含 n 的式子表示)。

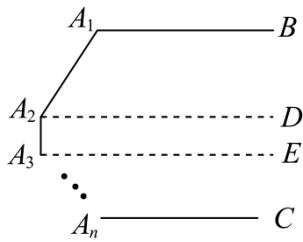


【答案】 $360^\circ/360$ 度 $(n-1) \cdot 180^\circ$

【分析】过点 A_2 向右作 $A_2D \parallel A_1B$ ，过点 A_3 向右作 $A_3E \parallel A_1B$ ，得到 $A_3E \parallel A_2D \parallel \dots \parallel A_1B \parallel A_nC$ ，根据两直

线平行同旁内角互补即可得出答案.

【详解】解:如图,过点 A_2 向右作 $A_2D \parallel A_1B$, 过点 A_3 向右作 $A_3E \parallel A_1B$,



$\therefore A_1B \parallel A_nC, \therefore A_3E \parallel A_2D \parallel \dots \parallel A_1B \parallel A_nC,$

$\therefore \angle A_1 + \angle A_1A_2D = 180^\circ, \angle DA_2A_3 + \angle A_2A_3E = 180^\circ, \dots,$

$\therefore \angle A_1 + \angle A_1A_2A_3 + \dots + \angle A_{n-1}A_nC = (n-1) \cdot 180^\circ,$

当 $n=3$ 时, $\angle A_1 + \angle A_2 + \angle A_3 = (3-1) \cdot 180^\circ = 360^\circ$ 故答案为: $360^\circ; (n-1) \cdot 180^\circ.$

【点睛】本题考查了平行线的判定和性质,根据题意作合适的辅助线是解题的关键.

模型3:牛角模型

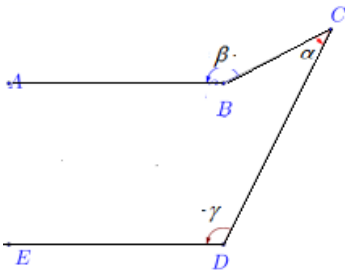


图1

如图1,已知: $AB \parallel DE$, 结论: $\alpha = \beta - \gamma.$

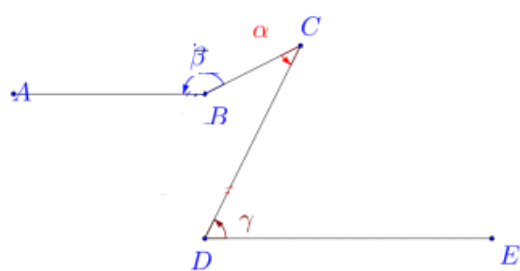
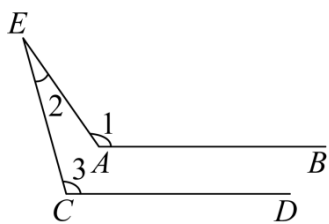


图2

如图2,已知: $AB \parallel DE$, 结论: $\alpha = \beta + \gamma - 180^\circ.$

例 12 (2023·安徽滁州·校联考二模) 如图,若 $AB \parallel CD$, 则 ()



A. $\angle 1 = \angle 2 + \angle 3$

B. $\angle 1 + \angle 3 = \angle 2$

C. $\angle 1 + \angle 2 + \angle 3 = 180^\circ$

D. $\angle 1 - \angle 2 + \angle 3 = 180^\circ$

【答案】A

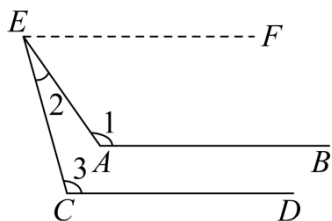
【分析】如图所示,过点 E 作 $EF \parallel AB$, 则 $AB \parallel CD \parallel EF$, 由平行线的性质得到 $\angle 3 + \angle CEF = 180^\circ, \angle 1 + \angle AEF = 180^\circ$, 进一步推出 $\angle 1 = \angle 2 + \angle 3$.

【详解】解:如图所示,过点 E 作 $EF \parallel AB$,

$\therefore AB \parallel CD, \therefore AB \parallel CD \parallel EF, \therefore \angle 3 + \angle CEF = 180^\circ, \angle 1 + \angle AEF = 180^\circ,$

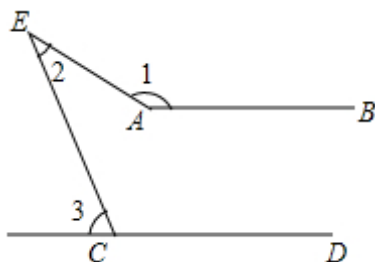
$\therefore \angle AEF = 180^\circ - \angle 1, \angle 3 = 180^\circ - \angle CEF = 180^\circ - \angle 2 - \angle AEF,$

$\therefore \angle 3 = 180^\circ - \angle 2 - 180^\circ + \angle 1, \therefore \angle 1 = \angle 2 + \angle 3$, 故选 A.



【点睛】本题主要考查了平行线的性质，熟知两直线平行，同旁内角互补是解题的关键。

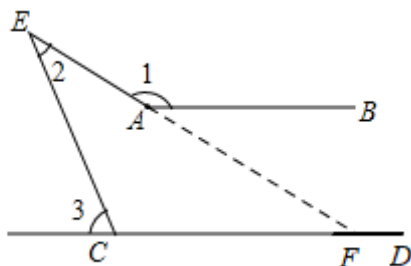
例 13 (2023·江苏·七年级假期作业) 如图，若 $AB \parallel CD$ ，则 $\angle 1 + \angle 3 - \angle 2$ 的度数为 _____



【答案】 180°

【分析】延长 EA 交 CD 于点 F ，则有 $\angle 2 + \angle EFC = \angle 3$ ，然后根据 $AB \parallel CD$ 可得 $\angle 1 = \angle EFD$ ，最后根据邻补角及等量代换可求解。

【详解】解：延长 EA 交 CD 于点 F ，如图所示：

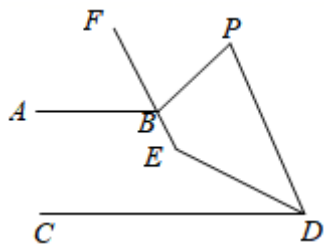


$\because AB \parallel CD, \therefore \angle 1 = \angle EFD, \because \angle 2 + \angle EFC = \angle 3, \therefore \angle EFC = \angle 3 - \angle 2,$

$\because \angle EFC + \angle EFD = 180^\circ, \therefore \angle 1 + \angle 3 - \angle 2 = 180^\circ$; 故答案为 180° 。

【点睛】本题主要考查三角形外角的性质及平行线的性质，熟练掌握三角形外角的性质及平行线的性质是解题的关键

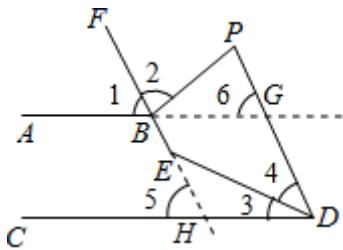
例 14 (2022·湖北洪山·七年级期中) 如图，已知 $AB \parallel CD$ ， P 为直线 AB, CD 外一点， BF 平分 $\angle ABP$ ， DE 平分 $\angle CDP$ ， BF 的反向延长线交 DE 于点 E ，若 $\angle FED = a$ ，试用 a 表示 $\angle P$ 为 _____。



【答案】 $\angle P = 360^\circ - 2a$

【分析】根据角平分线的性质得出 $\angle 1 = \angle 2, \angle 3 = \angle 4$ ，平行线的性质得出 $\angle 1 = \angle 5, \angle 6 = \angle PDC = 2\angle 3$ ，进而根据三角形内角和得出 $\angle 5, \angle FED$ ，再得到 $\angle P$ 和 a 的关系，然后即可用 a 表示 $\angle P$ 。

【详解】解:延长 AB 交 PD 于点 G , 延长 FE 交 CD 于点 H ,
 $\because BF$ 平分 $\angle ABP$, DE 平分 $\angle CDP$, $\therefore \angle 1 = \angle 2$, $\angle 3 = \angle 4$,
 $\because AB \parallel CD$, $\therefore \angle 1 = \angle 5$, $\angle 6 = \angle PDC = 2\angle 3$,
 $\because \angle PBG = 180^\circ - 2\angle 1$, $\therefore \angle PBG = 180^\circ - 2\angle 5$, $\therefore \angle 5 = 90^\circ - \frac{1}{2}\angle PBG$,
 $\because \angle FED = 180^\circ - \angle HED$, $\angle 5 = 180^\circ - \angle EHD$, $\angle EHD + \angle HED + \angle 3 = 180^\circ$,
 $\therefore 180^\circ - \angle 5 + 180^\circ - \angle FED + \angle 3 = 180^\circ$, $\therefore \angle FED = 180^\circ - \angle 5 + \angle 3$,
 $\therefore \angle FED = 180^\circ - (90^\circ - \frac{1}{2}\angle PBG) + \frac{1}{2}\angle 6 = 90^\circ + \frac{1}{2}(\angle PBG + \angle 6) = 90^\circ + \frac{1}{2}(180^\circ - \angle P) = 180^\circ - \frac{1}{2}\angle P$, $\therefore \angle FED = a$, $\therefore a = 180^\circ - \frac{1}{2}\angle P \therefore \angle P = 360^\circ - 2a$. 故答案为: $\angle P = 360^\circ - 2a$.



【点睛】此题考查了角平分线的性质和平行线的性质及三角形内角和,有一定的综合性,认真找出角的关系是关键.

- 例 15** (2023 春·广东深圳·九年级校校考期中) 已知直线 $AB \parallel CD$, 点 P 为直线 AB, CD 所确定的平面内的一点, (1) 问题提出: 如图 1, $\angle A = 120^\circ$, $\angle C = 130^\circ$. 求 $\angle APC$ 的度数;
 (2) 问题迁移: 如图 2, 写出 $\angle APC, \angle A, \angle C$ 之间的数量关系, 并说明理由;
 (3) 问题应用: 如图 3, $\angle EAH : \angle HAB = 1:3$, $\angle ECH = 20^\circ$, $\angle DCH = 60^\circ$, 求 $\frac{\angle H}{\angle E}$ 的值.

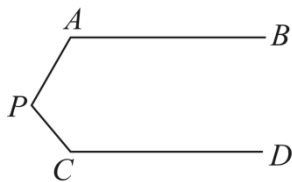


图1

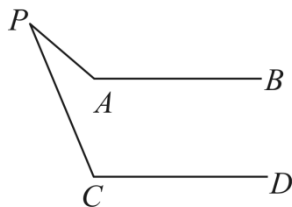


图2

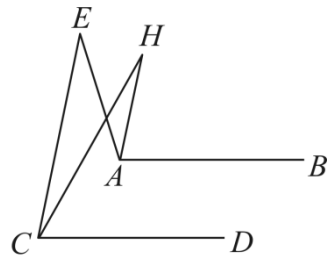


图3

【答案】(1) 110° (2) $\angle APC = \angle A - \angle C$, 理由见解析 (3) $\frac{3}{4}$

【分析】(1) 过点 P 作 $PQ \parallel AB$, 易得 $AB \parallel PQ \parallel CD$, 由平行线的性质可得 $\angle APQ = 60^\circ$, $\angle CPQ = 50^\circ$, 即可求出 $\angle APC$; (2) 过点 P 作 $PQ \parallel AB$, 易得 $AB \parallel PQ \parallel CD$, 根据平行线的性质可得 $\angle APC = \angle A - \angle C$;

(3) 过点 E 作 $EM \parallel AB$, 过点 H 作 $HN \parallel AB$, 易得 $EM \parallel CD$, $HN \parallel CD$, 根据平行线的性质可得 $\angle CEA = \angle BAE - \angle DCE$, $\angle CHA = \angle BAH - \angle DCH$, 再由已知等量代换, 即可求得 $\frac{\angle H}{\angle E}$ 的值.

【详解】(1) 解: 如图 1 所示, 过点 P 作 $PQ \parallel AB$, $\therefore \angle A + \angle APQ = 180^\circ$,

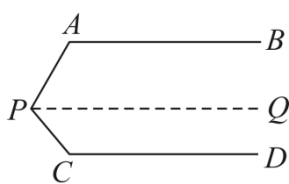


图1

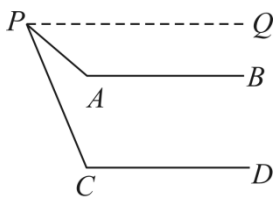


图2

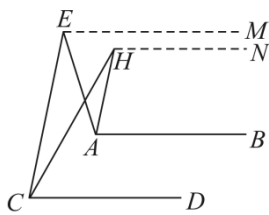


图3

$\because \angle A = 120^\circ, \therefore \angle APQ = 180^\circ - \angle A = 180^\circ - 120^\circ = 60^\circ, \because AB \parallel CD, \therefore PQ \parallel CD, \therefore \angle C + \angle CPQ = 180^\circ.$

$\because \angle C = 130^\circ, \therefore \angle CPQ = 180^\circ - \angle C = 180^\circ - 130^\circ = 50^\circ, \therefore \angle APC = \angle APQ + \angle CPQ = 60^\circ + 50^\circ = 110^\circ;$

(2) 解: $\angle APC = \angle A - \angle C$, 理由如下:

如图2, 过点P作 $PQ \parallel AB, \therefore \angle APQ = 180^\circ - \angle A, \because AB \parallel CD, \therefore PQ \parallel CD, \therefore \angle CPQ = 180^\circ - \angle C,$

$\therefore \angle APC = \angle CPQ - \angle APQ, \therefore \angle APC = 180^\circ - \angle C - (180^\circ - \angle A) = \angle A - \angle C;$

(3) 解: 如图3, 过点E作 $EM \parallel AB$, 过点H作 $HN \parallel AB$,

$\because AB \parallel CD, \therefore EM \parallel CD, HN \parallel CD,$

$\therefore \angle CEA = \angle CEM - \angle AEM = 180^\circ - \angle DCE - (180^\circ - \angle BAE) = \angle BAE - \angle DCE,$

$\angle CHA = \angle CHN - \angle AHN = 180^\circ - \angle DCH - (180^\circ - \angle BAH) = \angle BAH - \angle DCH,$

$\because \angle EAH : \angle HAB = 1 : 3, \angle ECH = 20^\circ, \angle DCH = 60^\circ,$

$\therefore \angle CEA = \angle BAE - \angle DCE = 4\angle EAH - 80^\circ, \angle CHA = \angle BAH - \angle DCH = 3\angle EAH - 60^\circ,$

$\therefore \frac{\angle CHA}{\angle CEA} = \frac{3\angle EAH - 60^\circ}{4\angle EAH - 80^\circ} = \frac{3(\angle EAH - 20^\circ)}{4(\angle EAH - 20^\circ)} = \frac{3}{4}.$

【点睛】本题考查了平行线的性质, 熟练掌握平行线的性质, 正确构造辅助线是解题的关键.

例 16 (2023·余干县八年级期末) 已知直线 $AB \parallel CD$, (1) 如图1, 直接写出 $\angle BME$ 、 $\angle E$ 、 $\angle END$ 的数量关系为 _____; (2) 如图2, $\angle BME$ 与 $\angle CNE$ 的角平分线所在的直线相交于点P, 试探究 $\angle P$ 与 $\angle E$ 之间的数量关系, 并证明你的结论; (3) 如图3, $\angle ABM = \frac{1}{n} \angle MBE, \angle CDN = \frac{1}{n} \angle NDE$, 直线MB、ND交于点F, 则 $\frac{\angle F}{\angle E} = \underline{\hspace{2cm}}.$

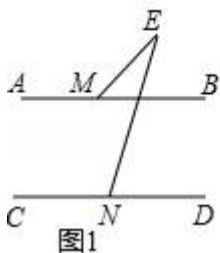


图1

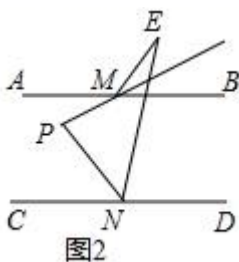


图2

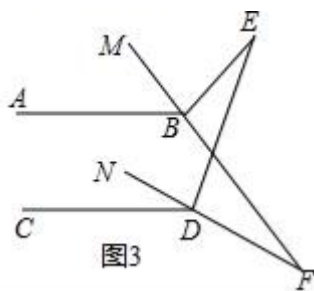


图3

【答案】(1) $\angle E = \angle END - \angle BME$ (2) $\angle E + 2\angle NPM = 180^\circ$ (3) $\frac{1}{n+1}$

【分析】(1) 根据平行线的性质和三角形外角定理即可解答.(2) 根据平行线的性质, 三角形外角定理, 角平分线的性质即可解答.(3) 根据平行线的性质和三角形外角定理即可解答.

【详解】(1) 如图1, $\because AB \parallel CD, \therefore \angle END = \angle EFB,$

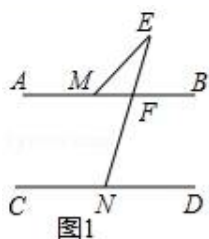


图1

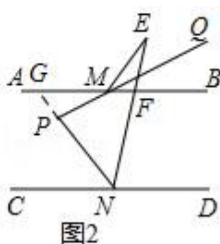


图2

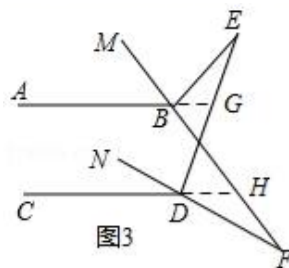


图3

$\because \angle EFB$ 是 $\triangle MEF$ 的外角, $\therefore \angle E = \angle EFB - \angle BME = \angle END - \angle BME$,

(2) 如图 2, $\because AB \parallel CD$, $\therefore \angle CNP = \angle NGB$,

$\because \angle NPM$ 是 $\triangle GPM$ 的外角, $\therefore \angle NPM = \angle NGB + \angle PMA = \angle CNP + \angle PMA$,

$\because MQ$ 平分 $\angle BME$, PN 平分 $\angle CNE$, $\therefore \angle CNE = 2\angle CNP$, $\angle FME = 2\angle BMQ = 2\angle PMA$,

$\because AB \parallel CD$, $\therefore \angle MFE = \angle CNE = 2\angle CNP$, $\therefore \triangle EFM$ 中, $\angle E + \angle FME + \angle MFE = 180^\circ$,

$\therefore \angle E + 2\angle PMA + 2\angle CNP = 180^\circ$, 即 $\angle E + 2(\angle PMA + \angle CNP) = 180^\circ$, $\therefore \angle E + 2\angle NPM = 180^\circ$;

(3) 如图 3, 延长 AB 交 DE 于 G , 延长 CD 交 BF 于 H ,

$\because AB \parallel CD$, $\therefore \angle CDG = \angle AGE$, $\because \angle ABE$ 是 $\triangle BEG$ 的外角,

$\therefore \angle E = \angle ABE - \angle AGE = \angle ABE - \angle CDE$, ①

$\because \angle ABM = \frac{1}{n} \angle MBE$, $\angle CDN = \frac{1}{n} \angle NDE$, $\therefore \angle ABM = \frac{1}{n+1} \angle ABE = \angle CHB$, $\angle CDN = \frac{1}{n+1} \angle CDE = \angle FDH$,

$\because \angle CHB$ 是 $\triangle DFH$ 的外角,

$\therefore \angle F = \angle CHB - \angle FDH = \frac{1}{n+1} \angle ABE - \frac{1}{n+1} \angle CDE = \frac{1}{n+1} (\angle ABE - \angle CDE)$, ②

由①代入②, 可得 $\angle F = \frac{1}{n+1} \angle E$, 即 $\frac{\angle F}{\angle E} = \frac{1}{n+1}$.

点睛: 本题考查了三角形外角定理, 平行线的性质, 角平分线的定义.

模型 4: 羊角模型

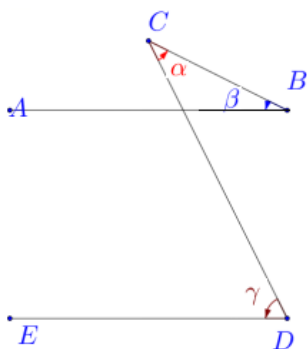


图 1

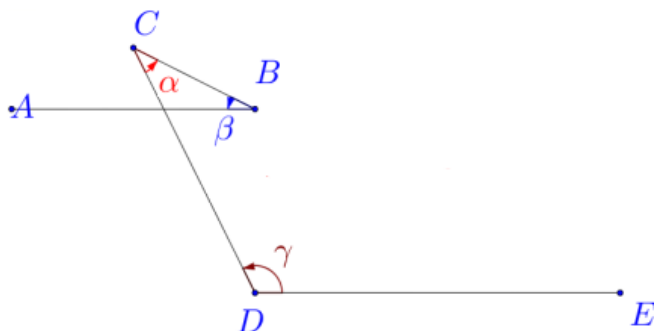


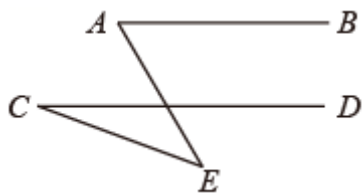
图 2

如图 1, 已知: $AB \parallel DE$, 结论: $\alpha = \gamma - \beta$.

如图 2, 已知: $AB \parallel DE$, 结论: $\alpha + \beta + \gamma = 180^\circ$.

例 17 (2023 春·上海·七年级专题练习) 如图所示, $AB \parallel CD$, $\angle E = 37^\circ$, $\angle C = 20^\circ$, 则 $\angle EAB$ 的度数为

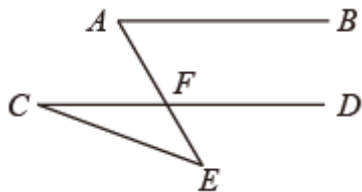
_____.



【答案】 57°

【分析】根据三角形内角和 180° 以及平行线的性质：1、如果两直线平行，那么它们的同位角相等；2、如果两直线平行，那么它们的同旁内角互补；3、如果两直线平行，那么它们的内错角相等，据此计算即可。

【详解】解：设AE、CD交于点F，

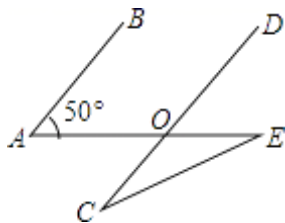


$\because \angle E = 37^\circ, \angle C = 20^\circ, \therefore \angle CFE = 180^\circ - 37^\circ - 20^\circ = 123^\circ, \therefore \angle AFD = 123^\circ,$

$\because AB \parallel CD, \therefore \angle AFD + \angle EAB = 180^\circ, \therefore \angle EAB = 180^\circ - 123^\circ = 57^\circ,$ 故答案为： 57° 。

【点睛】本题主要考查三角形内角和定理以及平行线的性质，熟知平行的性质是解题的关键。

例 18 (2022·江苏七年级期中) 如图所示，已知 $AB \parallel CD, \angle A = 50^\circ, \angle C = \angle E$ 。则 $\angle C$ 等于 ()



A. 20°

B. 25°

C. 30°

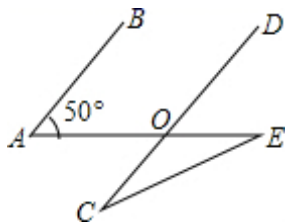
D. 40°

【答案】B

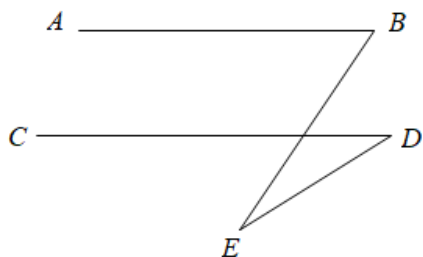
【分析】根据 $AB \parallel CD, \angle A = 50^\circ$ ，所以 $\angle A = \angle AOC$ 。又因为 $\angle C = \angle E, \angle AOC$ 是外角，所以可求得 $\angle C$ 。

【详解】解： $\because AB \parallel CD, \angle A = 50^\circ, \therefore \angle A = \angle AOC$ (内错角相等)，

又 $\because \angle C = \angle E, \angle AOC$ 是外角， $\therefore \angle C = 50^\circ \div 2 = 25^\circ$ 。故选B。



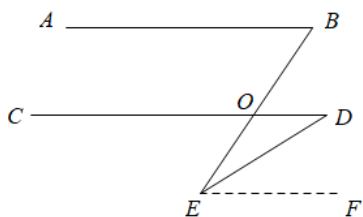
例 19 (2023春·浙江·七年级专题练习) 已知 $AB \parallel CD$ ，求证： $\angle B = \angle E + \angle D$



【答案】见解析

【分析】过点 E 作 $EF \parallel CD$, 根据平行线的性质即可得出 $\angle B = \angle BOD$, 根据平行线的性质即可得出 $\angle BOD = \angle BEF$ 、 $\angle D = \angle DEF$, 结合角之间的关系即可得出结论.

【详解】证明: 过点 E 作 $EF \parallel CD$, 如图



$\because AB \parallel CD, \therefore \angle B = \angle BOD, \because EF \parallel CD$ (辅助线),

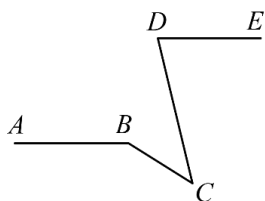
$\therefore \angle BOD = \angle BEF$ (两直线平行, 同位角相等); $\angle D = \angle DEF$ (两直线平行, 内错角相等);

$\therefore \angle BEF = \angle BED + \angle DEF = \angle BED + \angle D$ (等量代换),

$\therefore \angle BOD = \angle E + \angle D$ (等量代换), 即 $\angle B = \angle E + \angle D$.

【点睛】本题考查了平行线的性质以及角的计算, 解题的关键是根据平行线的性质找出相等或互补的角.

例 20 (2023·河南·统考三模) 如图, 已知 $AB \parallel DE$, $\angle ABC = 150^\circ$, $\angle CDE = 75^\circ$, 则 $\angle BCD$ 的度数为 ()



A. 55°

B. 60°

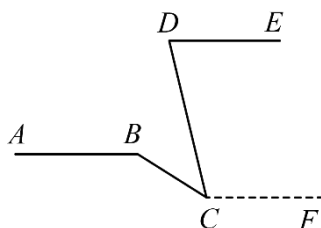
C. 45°

D. 50°

【答案】C

【分析】过点 C 作 $CF \parallel AB$, 则 $AB \parallel DE \parallel CF$, 根据平行线的性质可得到 $\angle BCF = \angle ABC = 150^\circ$, $\angle DCF = 180^\circ - \angle CDE = 105^\circ$, 即可求得 $\angle BCD = \angle BCF - \angle DCF = 45^\circ$.

【详解】如图, 过点 C 作 $CF \parallel AB$, $\angle DCF + \angle CDE = 180^\circ$



$\because AB \parallel DE, CF \parallel AB, \therefore AB \parallel DE \parallel CF$.

$\therefore \angle BCF = \angle ABC = 150^\circ, \therefore$

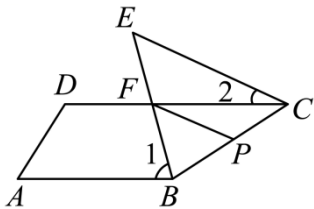
$\because \angle CDE = 75^\circ, \therefore \angle DCF = 180^\circ - 75^\circ = 105^\circ$.

$\therefore \angle BCD = \angle BCF - \angle DCF = 150^\circ - 105^\circ = 45^\circ$. 故选 C.

【点睛】本题主要考查了平行线的性质,正确作出辅助线,利用平行线的性质求解是解决问题的关键.

例 21 (2023·河北沧州·校考模拟预测) 如图, $\angle A = 58^\circ$, $\angle D = 122^\circ$, $\angle 1 = 3\angle 2$, $\angle 2 = 25^\circ$, 点 P 是 BC 上一点.

(1) $\angle DFE$ 的度数为 _____; (2) 若 $\angle BFP = 50^\circ$. 则 CE 与 PF _____ (填“平行”或“不平行”).



【答案】 $75^\circ/75$ 度 平行

【分析】(1) 根据同旁内角的判定可得 $AB \parallel CD$, 根据平行线的性质可得 $\angle DFE$ 的度数;

(2) 根据对顶角相等可得 $\angle CFP$ 的度数, 根据同位角的判定可得 $CE \parallel PF$.

【详解】解: (1) $\because \angle A = 58^\circ, \angle D = 122^\circ, \therefore \angle A + \angle D = 180^\circ, \therefore AB \parallel CD,$

$\because \angle 1 = 3\angle 2, \angle 2 = 25^\circ, \therefore \angle 1 = 75^\circ, \therefore \angle DFE = \angle 1 = 75^\circ;$ 故答案为: 75° .

(2) $\because \angle DFE = 75^\circ, \therefore \angle CFB = 75^\circ,$

$\because \angle BFP = 50^\circ, \therefore \angle CFP = 25^\circ, \therefore \angle CFP = \angle 2, \therefore CE \parallel PF.$ 故答案为: 平行.

【点睛】本题考查了对顶角相等, 平行线的判定和性质, 熟练掌握平行线的判定和性质是解题的关键.

模型 5: 蛇形模型 (“5”字模型)

基本模型: 如图, $AB \parallel CD$, 结论: $\angle 1 + \angle 3 - \angle 2 = 180^\circ$.

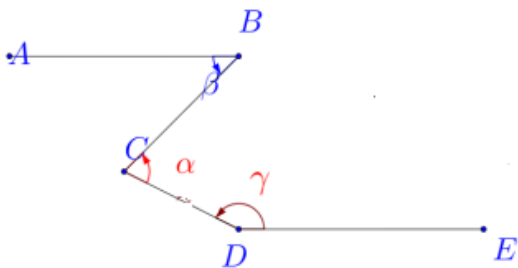


图 1

如图 1, 已知: $AB \parallel DE$, 结论: $\alpha = \beta + 180^\circ - \gamma$.

如图 2, 已知: $AB \parallel DE$, 结论: $\alpha = \gamma + 180^\circ - \beta$.

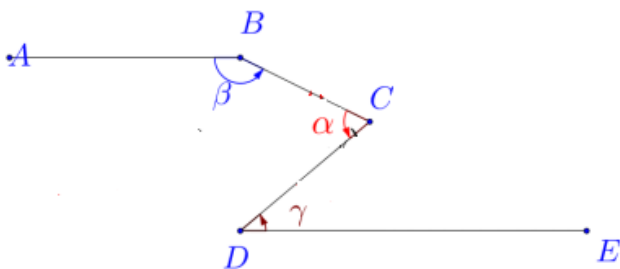
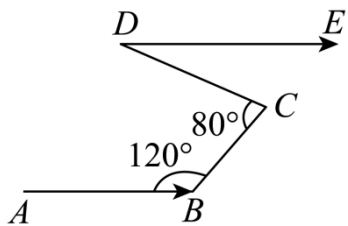


图 2

例 22 (2023·四川广元·统考三模) 珠江流域某江段江水流向经过 B 、 C 、 D 三点, 拐弯后与原来方向相同, 如图, 若 $\angle ABC = 120^\circ$, $\angle BCD = 80^\circ$, 则 $\angle CDE$ 等于 ()

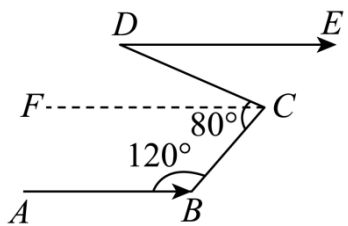


- A. 50° B. 40° C. 30° D. 20°

【答案】D

【分析】过点C作 $CF \parallel AB$,根据平行线的性质即可求出 $\angle CDE$ 的度数.

【详解】解:过点C作 $CF \parallel AB$, $\therefore \angle ABC + \angle BCF = 180^\circ$,



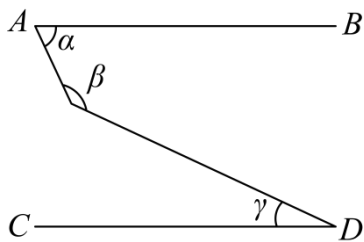
$$\because \angle ABC = 120^\circ, \therefore \angle BCF = 180^\circ - \angle ABC = 180^\circ - 120^\circ = 60^\circ;$$

$$\because \angle BCD = 80^\circ, \therefore \angle DCF = 80^\circ - \angle BCF = 80^\circ - 60^\circ = 20^\circ;$$

由题意 $DE \parallel AB$, $\therefore CF \parallel DE$, $\therefore \angle CDE = \angle DCF = 20^\circ$. 故选: D

【点睛】本题考查平行线的判断和性质,作出辅助线,灵活运用平行线的性质是解题的关键.

例 23 (2023·湖南长沙·九年级校联考期中) 如图,若 $AB \parallel CD$, $\angle \alpha = 65^\circ$, $\angle \gamma = 25^\circ$,则 $\angle \beta$ 的度数是 ()

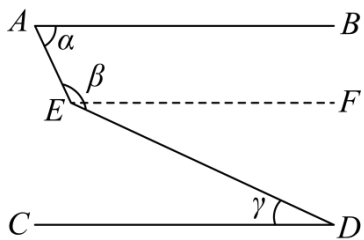


- A. 115° B. 130° C. 140° D. 150°

【答案】C

【分析】利用平行线的传递性作出辅助线EF,再通过平行线的性质即可解决问题.

【详解】解:过E作AB的平行线EF,如图所示;

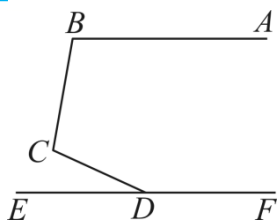


$$\therefore \angle AEF = 180^\circ - \angle \alpha = 180^\circ - 65^\circ = 115^\circ, \because AB \parallel CD \therefore EF \parallel CD \therefore \angle FED = \angle \gamma = 25^\circ$$

$$\therefore \angle \beta = \angle AEF + \angle FED = 115^\circ + 25^\circ = 140^\circ \text{ 故选 } C.$$

【点睛】本题考查了平行线的基本性质与平行的传递性,两直线平行,内错角相等、同旁内角互补,根据传递性做出辅助线是解决问题的关键.

例 24 (2023·河南周口·校联考三模) 如图, $AB \parallel EF$, $\angle B = 100^\circ$, $\angle CDE = 25^\circ$, 则 $\angle BCD$ 的度数是 ()

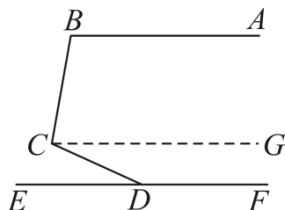


- A. 125° B. 75° C. 95° D. 105°

【答案】D

【分析】作 $CG \parallel EF$, 则 $CG \parallel AB \parallel EF$, 根据平行线的性质分别求出 $\angle GCD$ 和 $\angle BCG$, 则 $\angle BCD = \angle GCD + \angle BCG = 105^\circ$.

【详解】解: 如图, 作 $CG \parallel EF$, 则 $CG \parallel AB \parallel EF$,



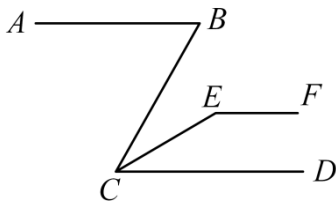
$\because CG \parallel EF, \therefore \angle GCD = \angle CDE = 25^\circ, \because CG \parallel AB, \therefore \angle B + \angle BCG = 180^\circ,$

$\therefore \angle BCG = 180^\circ - \angle B = 180^\circ - 100^\circ = 80^\circ,$

$\therefore \angle BCD = \angle GCD + \angle BCG = 25^\circ + 80^\circ = 105^\circ$ 故选 D.

【点睛】本题考查根据平行线的性质求角的度数, 解题的关键是正确添加辅助线.

例 25 (2023·陕西西安·校考模拟预测) 如图, $AB \parallel CD, CD \parallel EF, CE$ 平分 $\angle BCD$, 若 $\angle ABC = 58^\circ$, 则 $\angle CEF$ 的度数为 ()



- A. 131° B. 141° C. 151° D. 161°

【答案】C

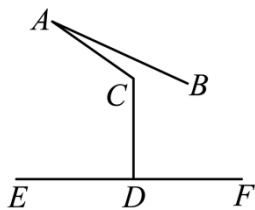
【分析】由平行线的性质可知 $\angle BCD = \angle ABC = 58^\circ, \angle CEF + \angle DCE = 180^\circ$. 再由角平分线的定义即可求解.

【详解】 $\because AB \parallel CD, \therefore \angle BCD = \angle ABC = 58^\circ. \because CE$ 平分 $\angle BCD, \therefore \angle DCE = \frac{1}{2} \angle BCD = 29^\circ.$

$\because CD \parallel EF, \therefore \angle CEF + \angle DCE = 180^\circ, \therefore \angle CEF = 180^\circ - \angle DCE = 151^\circ.$ 故选: C.

【点睛】本题考查平行线的性质, 角平分线的定义. 利用数形结合的思想是解题关键.

例 26 (2023·江西·九年级校考阶段练习) 如图 $\angle BAC = 10^\circ, \angle ACD = 125^\circ, CD \perp EF$ 于点 D, 将 AB 绕点 A 逆时针旋转 α , 使 $AB \parallel EF$, 则 α 的最小值为 _____.



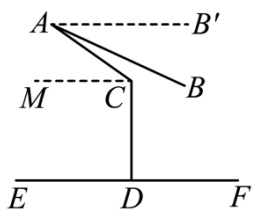
【答案】 $25^\circ/25$ 度

【分析】过点 C 作 $CM \parallel EF$, 过点 A 作 $AB' \parallel EF$, 利用平行线的性质即可求解.

【详解】解: 如图, 过点 C 作 $CM \parallel EF$, 则 $\angle MCD = 180^\circ - \angle CDE = 90^\circ$,

$\therefore \angle ACM = \angle ACD - \angle MCD = 35^\circ$. 过点 A 作 $AB' \parallel EF$, 则 $\angle B'AC = \angle ACM = 35^\circ$.

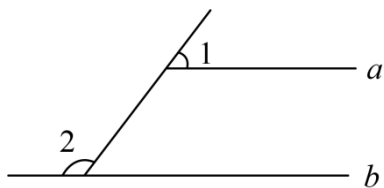
$\therefore \angle BAB' = 35^\circ - 10^\circ = 25^\circ$, 故 α 的最小值为 25° . 故答案为: 25°



【点睛】本题考查了平行线的性质, 掌握两直线平行, 同旁内角互补; 两直线平行, 内错角相等是解题的关键.

课后专项训练

题目 1 (2023·山东临沂·统考二模) 如图, $a \parallel b$, $\angle 1 = 45^\circ$, 则 $\angle 2$ 的度数为 ()



A. 105°

B. 125°

C. 135°

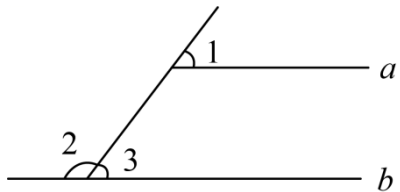
D. 145°

【答案】C

【分析】先根据平行线的性质可得 $\angle 3 = \angle 1 = 45^\circ$, 再根据邻补角的定义即可得.

【详解】解: 如图, $\because a \parallel b$, $\angle 1 = 45^\circ$, $\therefore \angle 3 = \angle 1 = 45^\circ$,

$\therefore \angle 2 = 180^\circ - \angle 3 = 135^\circ$, 故选: C.



【点睛】本题考查了平行线的性质、邻补角, 熟练掌握平行线的性质是解题关键.

题目 2 (2023春·安徽·九年级专题练习) 如图, 已知: $AB \parallel EF$, $\angle B = \angle E$, 求证: $BC \parallel DE$. 在证明该结论时, 需添加辅助线, 则以下关于辅助线的作法不正确的是 ()

以上内容仅为本文档的试下载部分，为可阅读页数的一半内容。如要下载或阅读全文，请访问：<https://d.book118.com/887150066156006031>