Evolutionary Game Theory

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CIS620

Outline

- EGT versus CGT
- Evolutionary Stable Strategies
  - Concepts and Examples
- Replicator Dynamics
  - Concepts and Examples
- Overview of 2 papers
  - Selection methods, finite populations

EGT v. Conventional Game Theory

- Models used to study interactive decision making.
- Equilibrium is still at heart of the model.
- Key difference is in the notion of rationality of agents.

Agent Rationality

- In GT, one assumes that agents are perfectly rational.
- In EGT, trial and error process gives strategies that can be selected for by some force (evolution - biological, cultural, etc…).
- This lack of rationality is the point of departure between EGT and GT.

Evolution

- When in biological sense, natural selection is mode of evolution.
- Strategies that increase Darwinian fitness are preferable.
- Frequency dependent selection.

Evolutionary Game Theory (EGT)

Has origins in work of R.A. Fisher [The Genetic Theory of Natural Selection (1930)].

•Fisher studied why sex ratio is approximately equal in many species.

•Maynard Smith and Price introduce concept of an ESS [The Logic of Animal Conflict (1973)].

•Taylor, Zeeman, Jonker (1978-1979) provide continuous dynamics for EGT (replicator dynamics).

ESS Approach

- ESS = Nash Equilibrium + Stability Condition
- Notion of stability applies only to isolated bursts of mutations.
- Selection will tend to lead to an ESS, once at an ESS selection keeps us there.

ESS - Definition

•Consider a 2 player symmetric game with ESS given by I with payoff matrix E.

•Let p be a small percentage of population playing mutant strategy $J \neq I$.

•Fitness given by

$$W(I) = W_0 + (1\text{-}p)E(I,I) + pE(I,J)$$
$$W(J) = W_0 + (1\text{-}p)E(J,I) + pE(J,J)$$

•Require that $W(I) > W(J)$

ESS - Definition

- Standard Definition for ESS (Maynard Smith).
- I is an ESS if for all $J \neq I$,
  $E(I,I) \geq E(J,I)$ and
  $E(I,I) = E(J,I) \Rightarrow E(I,J) > E(J,J)$
  where E is the payoff function .

ESS - Definition

Assumptions:

1) Pairwise, symmetric contests

2) Asexual inheritance

3) Infinite population

4) Complete mixing

ESS - Existence

- Let G be a two-payer symmetric game with 2 pure strategies such that

  $E(s1,s1) \neq E(s2,s1)$ <u>AND</u>
  $E(s1,s2) \neq E(s2,s2)$

  then G has an ESS.

ESS Existence

- If a > c, then s1 is ESS.
- If d > b, then s2 is ESS.

	s1	s2
s1	a	b
s2	c	d

- Otherwise, ESS given by playing s1 with probability equal to (b-d)/[(b-d)+(a-c)].

ESS - Example 1

- Consider the Hawk-Dove game with payoff matrix

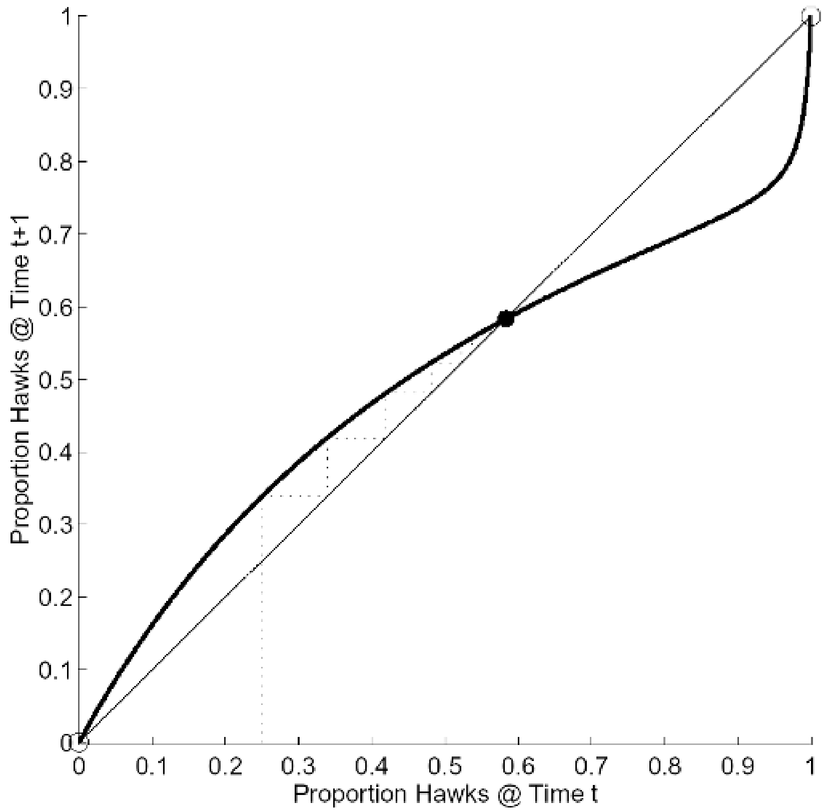
	H	D
H	"-25,-25"	50,0
D	0,50	15,15

- Nash equilibrium given by (7/12,5/12).
- This is also an ESS.

ESS - Example 1

- Bishop-Cannings Theorem:
  If I is a mixed ESS with support a,b,c,…, then E(a,I) = E(b,I) = … = E(I,I).
- At a stable polymorphic state, the fitness of Hawks and Doves must be the same.
- W(H) = W(D) => The ESS given is a stable polymorphism.

Stable Polymorphic State



ESS - Example 2

- Consider the Rock-Scissors-Paper Game.
- Payoff matrix is given by

	R	S	P
R	-e	1	-1
S	-1	-e	1
P	1	-1	-e

- Then I = (1/3,1/3,1/3) is an ESS but stable polymorphic population 1/3R,1/3P,1/3S is not stable.

ESS - Example 3

- Payoff matrix :

	s1	s2	s3
s1	1,1	2,-2	"-2,2"
s2	"-2,2"	1,1	2,-2
s3	2,-2	"-2,2"	1,1

- Then I = (1/3,1/3,1/3) is the unique NE, but not an ESS since E(I,s1)=E(s1,s1)= 1.

Sex Ratios

- Recall Fisher's analysis of the sex ratio.
- Why are there approximately equal numbers of males and females in a population?
- Greatest production of offspring would be achieved if there were many times more females than males.

Sex Ratios

- Let sex ratio be s males and (1-s) females.
- W(s,s′) = fitness of playing s in population of s′
- Fitness is the number of grandchildren
- $W(s,s') = N^2[(1-s) + s(1-s')/s']$
  $W(s',s') = 2N^2(1-s')$
- Need s* s.t. $\forall s\ W(s^*,s^*) \geq W(s,s^*)$

Dynamics Approach

- Aims to study actual evolutionary process.
- One Approach is Replicator Dynamics.
- Replicator dynamics are a set of deterministic difference or differential equations.

RD - Example 1

- Assumptions: Discrete time model, non-overlapping generations.
- $x_i(t)$ = proportion playing i at time t
- $\pi(i,x(t))$ = E(number of replacement for agent playing i at time t)
- $\Sigma_j \{x_j(t)\ \pi(j,x(t))\} = v(x(t))$
- $x_i(t+1) = [x_i(t)\ \pi(i,x(t))]/\ v(x(t))$

RD - Example 1

- Assumptions: Discrete time model, non-overlapping generations.

- $$x_i(t+1) - x_i(t) = \frac{x_i(t) \quad [\pi(i,x(t)) - v(x(t))]}{v(x(t))}$$

RD - Example 2

- Assumptions : overlapping generations, discrete time model.
- In time period of length τ, let fraction τ give birth to agents also playing same strategy.
- $\Sigma_j \; x_j(t)[1 + \tau \, \pi(j,x(t))] = v(x(t))$
- $x_i(t+\tau) = \dfrac{x_i(t)[1 + \tau \, \pi(i,x(t))]}{v(x(t))}$

RD - Example 2

- Assumptions : overlapping generations, discrete time model.

- $x_i(t+\tau) - x_i(t) = \dfrac{x_i(t)[\tau\ \pi(i,x(t)) - \blacktriangle\ v(x(t))]}{1+ \tau\ v(x(t))}$

以上内容仅为本文档的试下载部分，为可阅读页数的一半内容。如要下载或阅读全文，请访问：https://d.book118.com/906153125150011004