梁的内力

梁的内力

§ 1 弯曲的概念和工程实例

§ 2 静定梁的分类

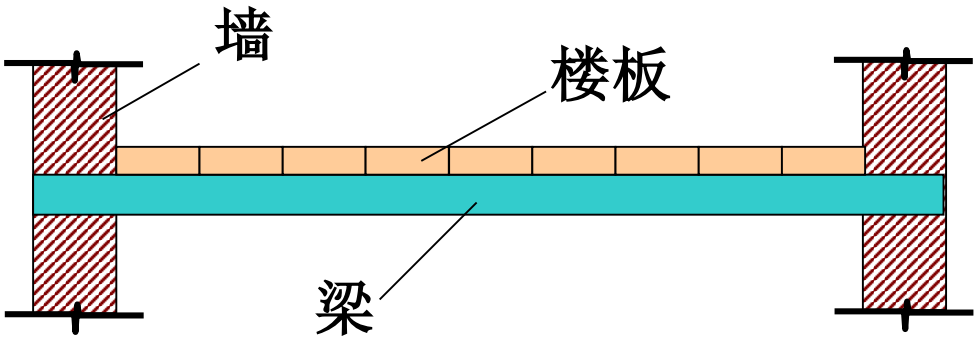
§ 3 剪力与弯矩 剪力图与弯矩图

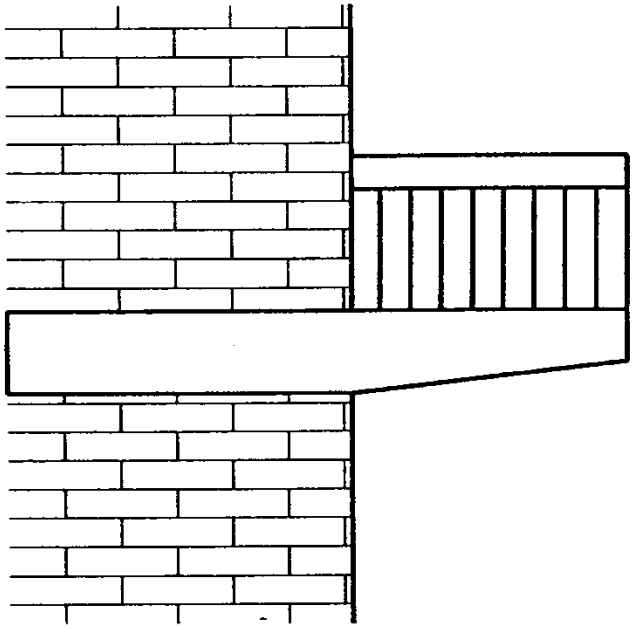
§ 4 剪力、弯矩和分布荷载集度间的关系

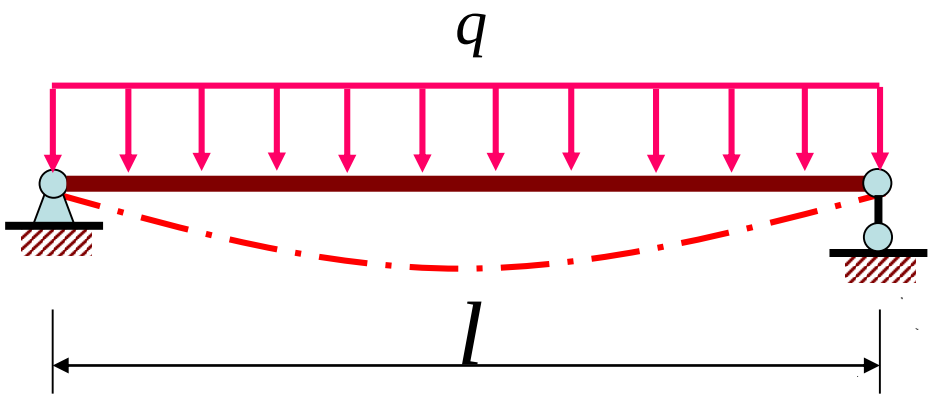
§ 5 按叠加原理作弯矩图

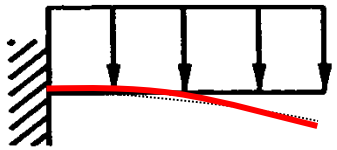
§ 1 概念和工程实例

※ 工程应用



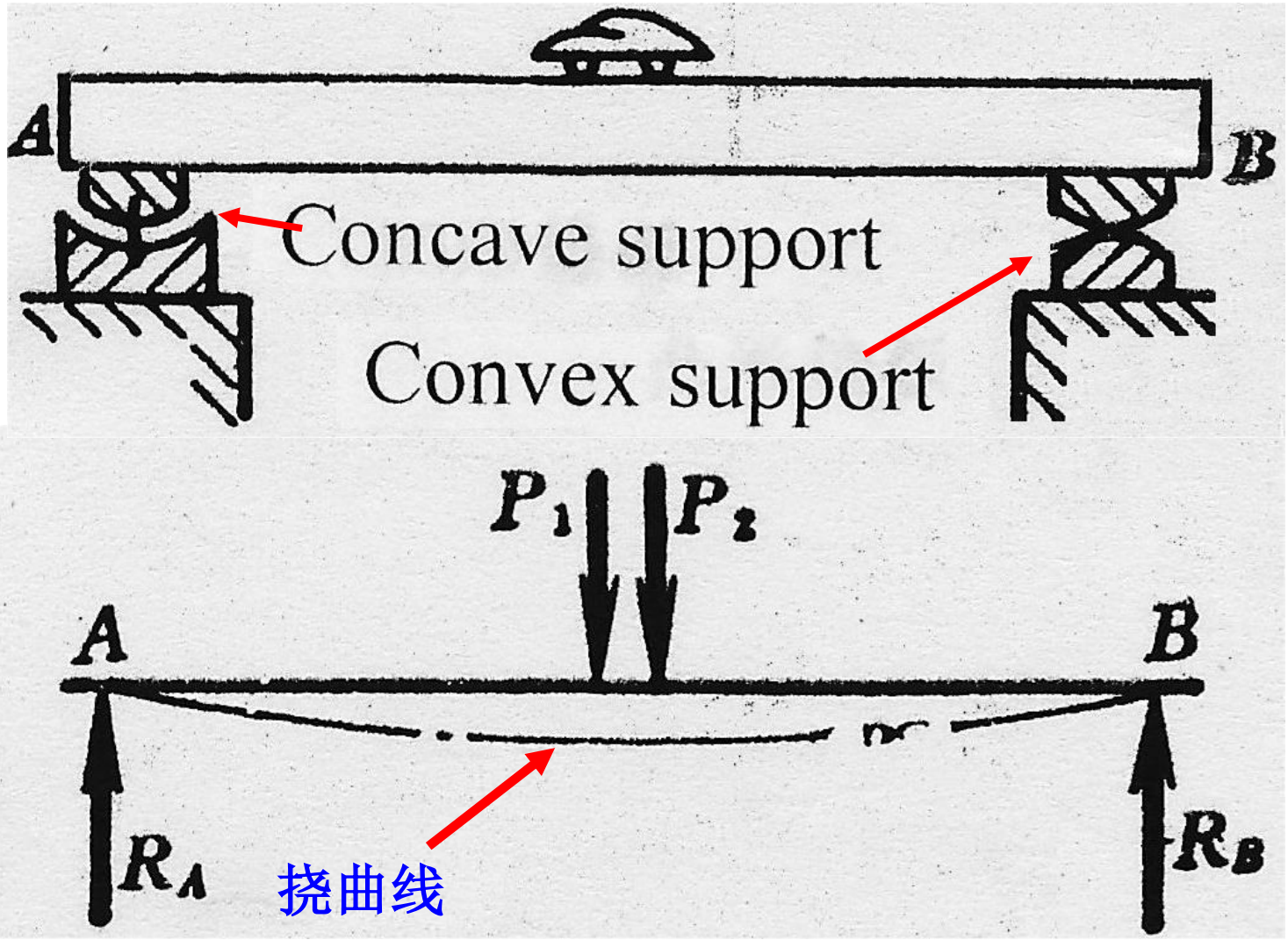






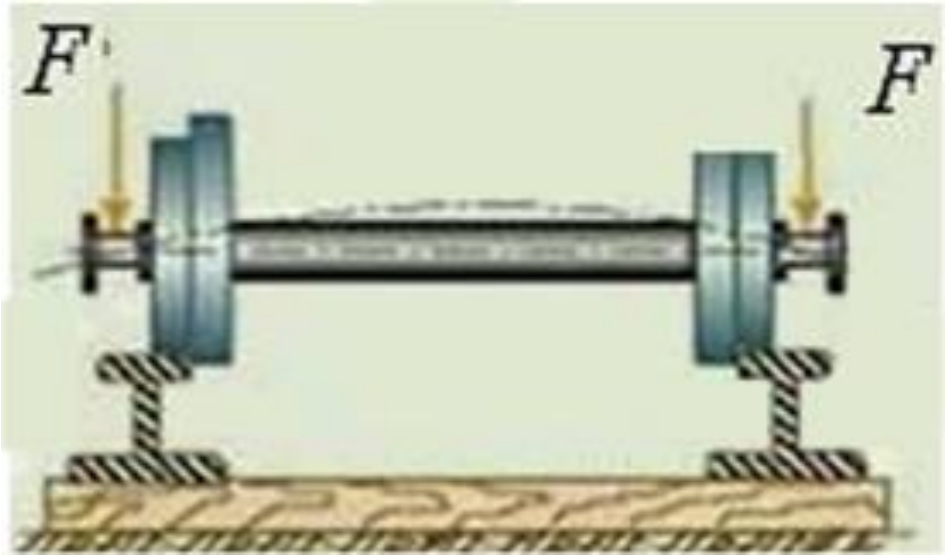
§ 1 概念和工程实例

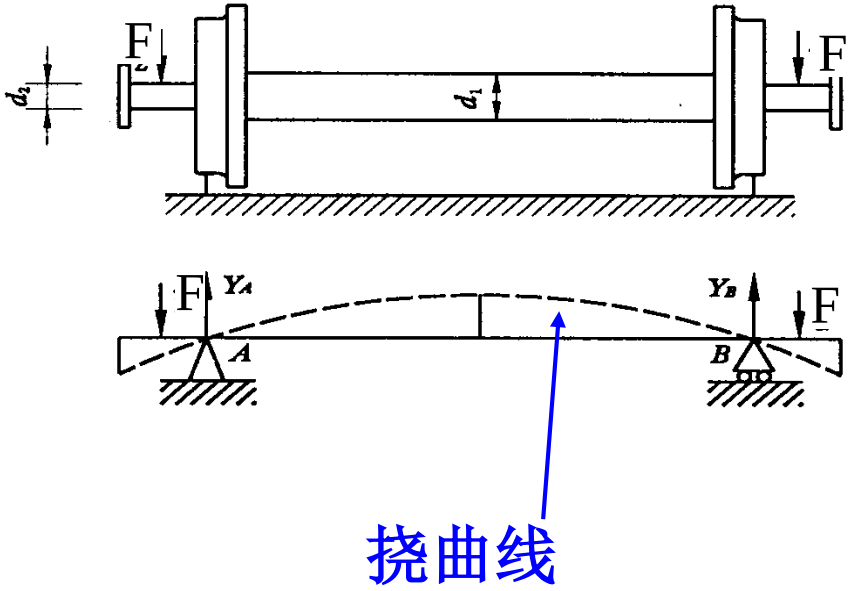
※ 工程应用



§ 1 概念和工程实例

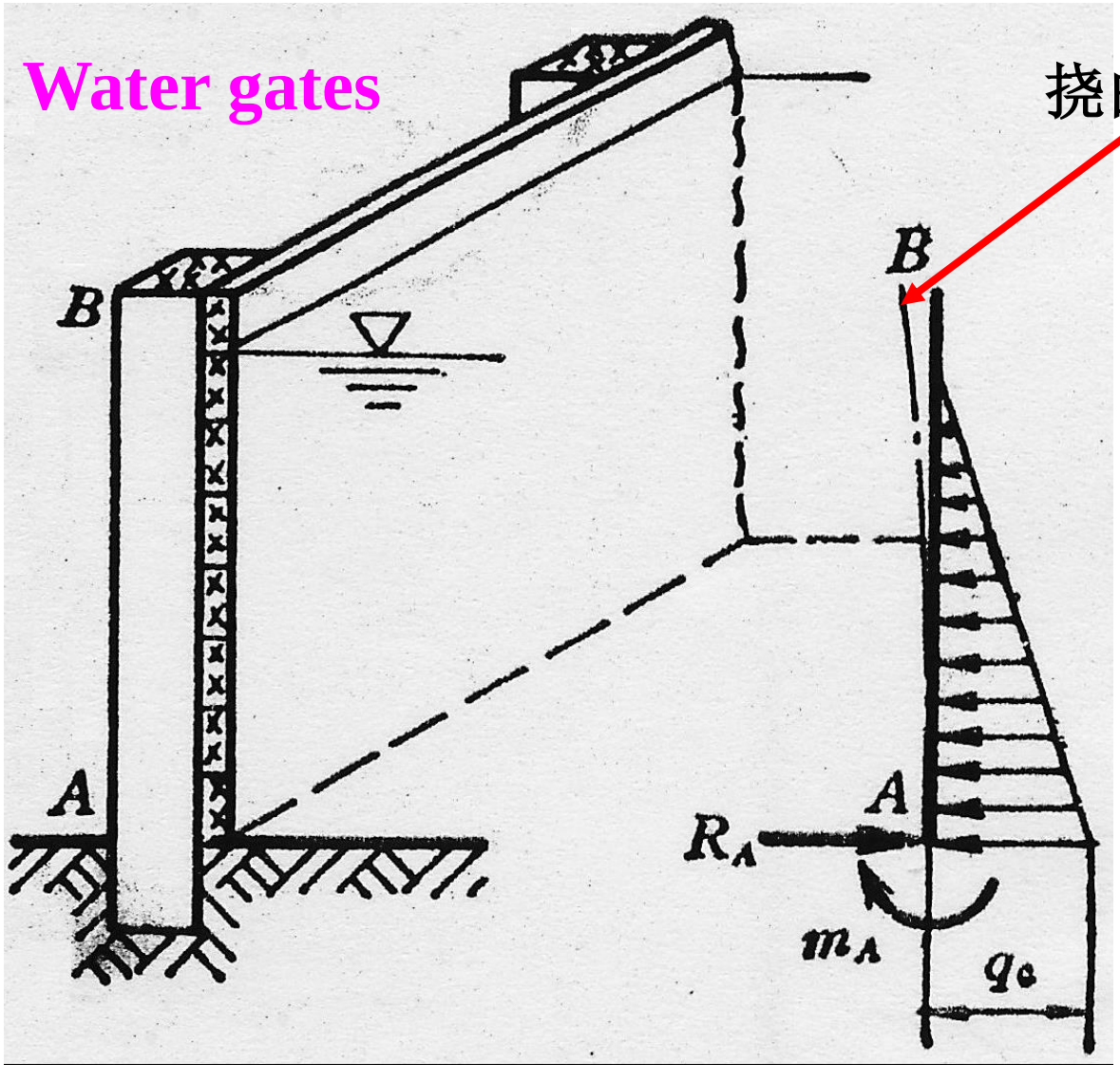
※ 工程应用





§ 1 概念和工程实例

※ 工程应用



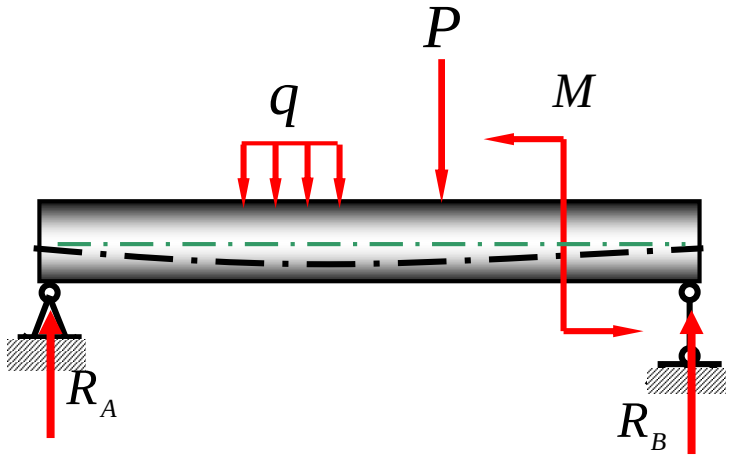
§ 1 概念和工程实例

※ 定义

若构件承受垂直于轴线的纵向载荷，则构件发生弯曲变形.

弯曲变形后, 构件的初始直线轴线变成的曲线，称为挠曲线.

主要发生弯曲变形的构件称为 梁.

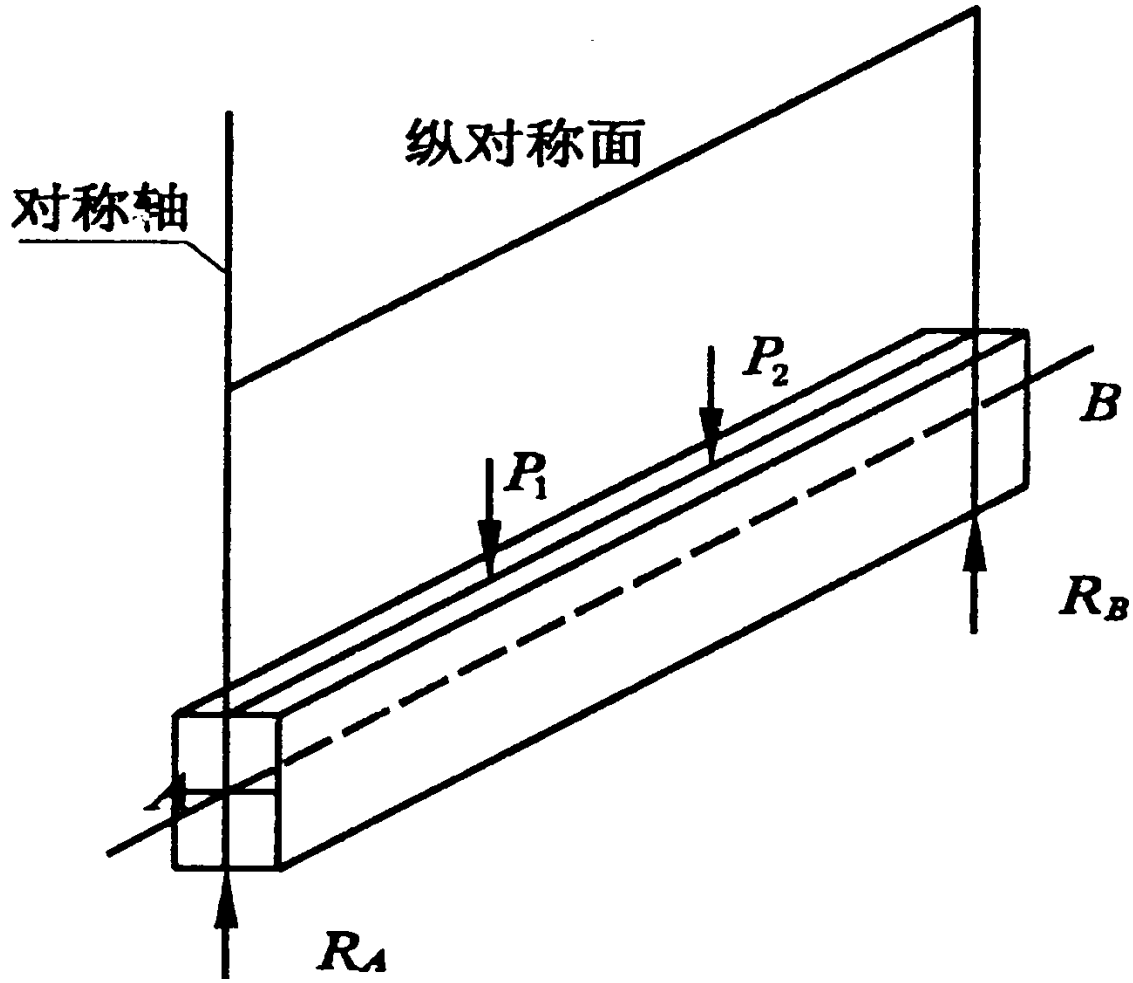


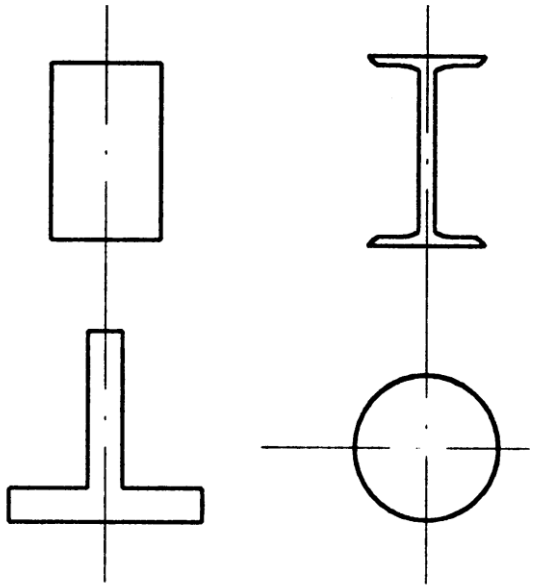
受力特点——作用于杆件上的外力都垂直于杆的轴线。

变形特点——杆轴线由直线变为一条平面的曲线。

§ 1 概念和工程实例

※ 定义





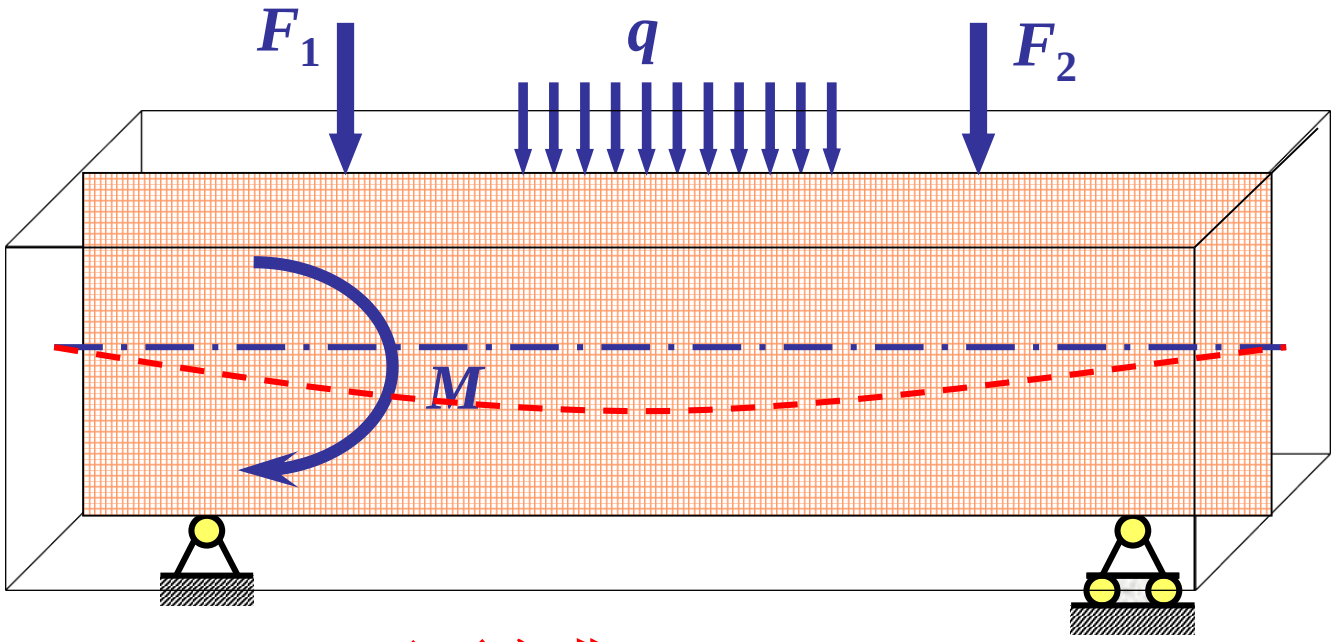
平面弯曲

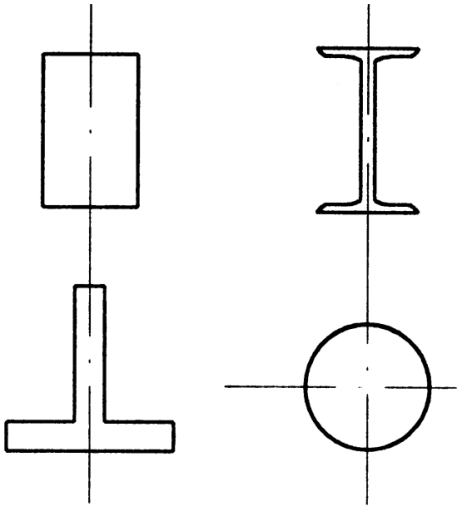
§ 1 概念和工程实例

※ 平面弯曲的特点

受力特点： 外力垂直于杆轴且在梁的纵向对称面内（通过或平行形心主轴上且过弯曲中心）

变形特点: 杆轴线在纵向对称面内由直线变成平面曲线





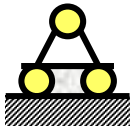
平面弯曲

§ 2 静定梁的分类

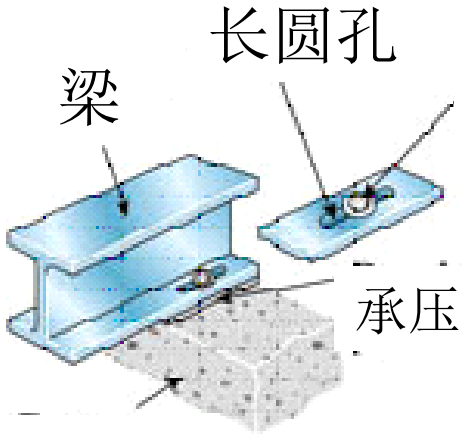
※ 支座类型

铰支座：

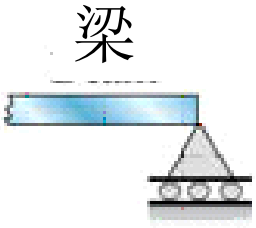
滑动铰支座



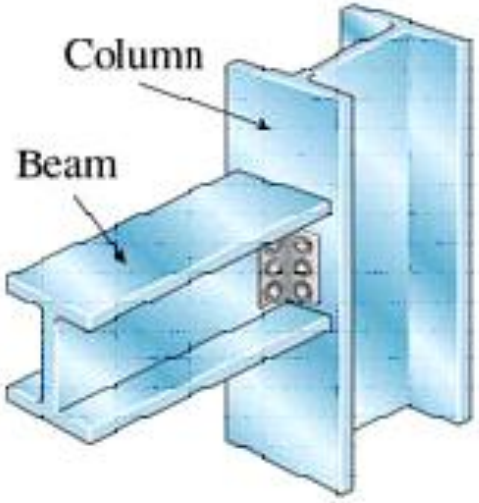
固定支座



(a)

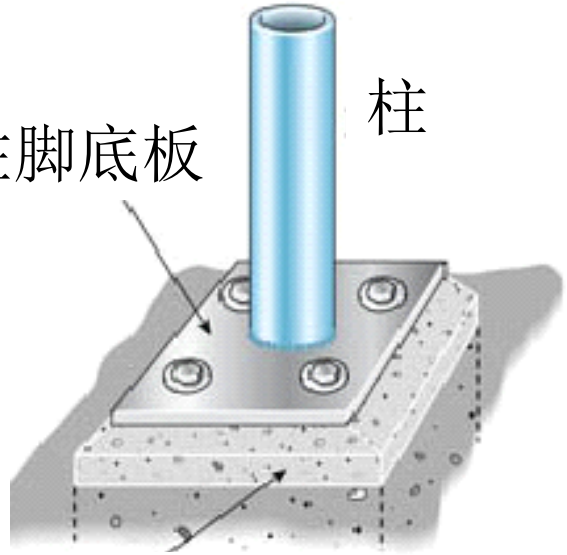


(b)

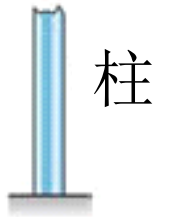


(c)

(d)



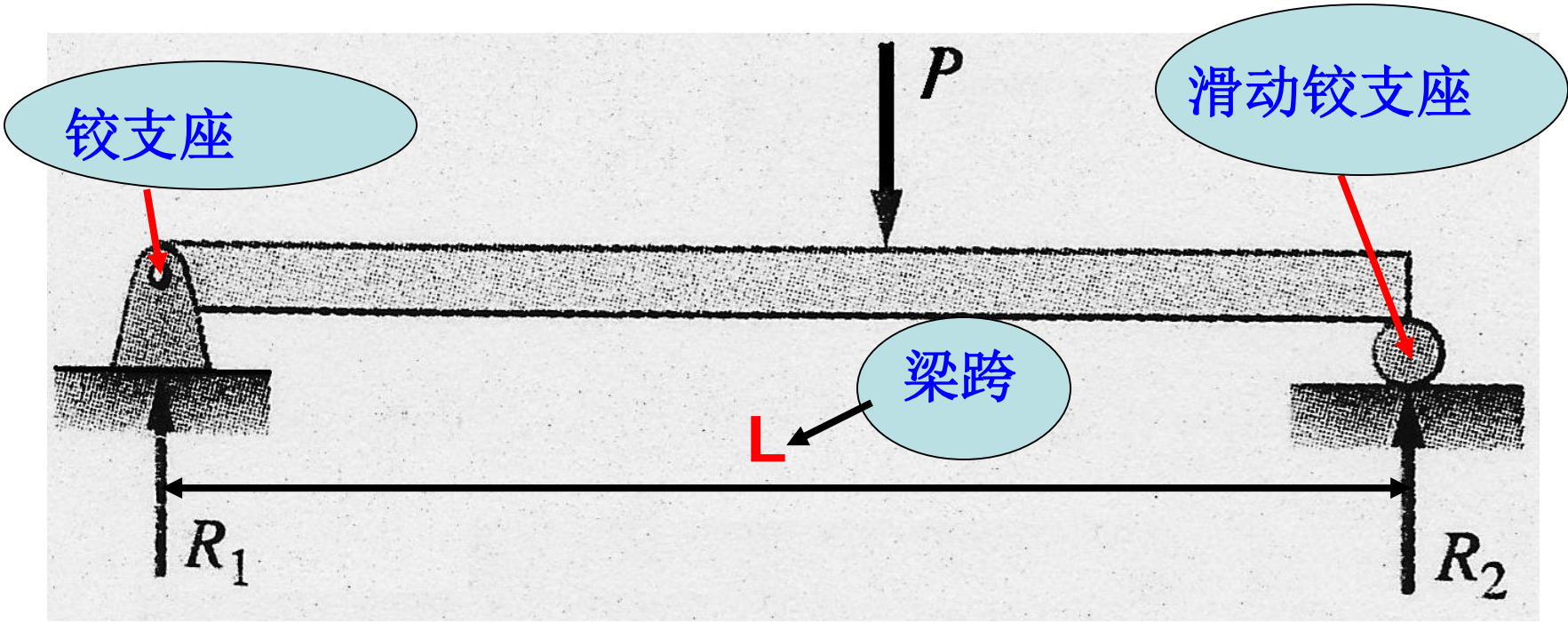
(e)



(f)

§ 2 静定梁的分类

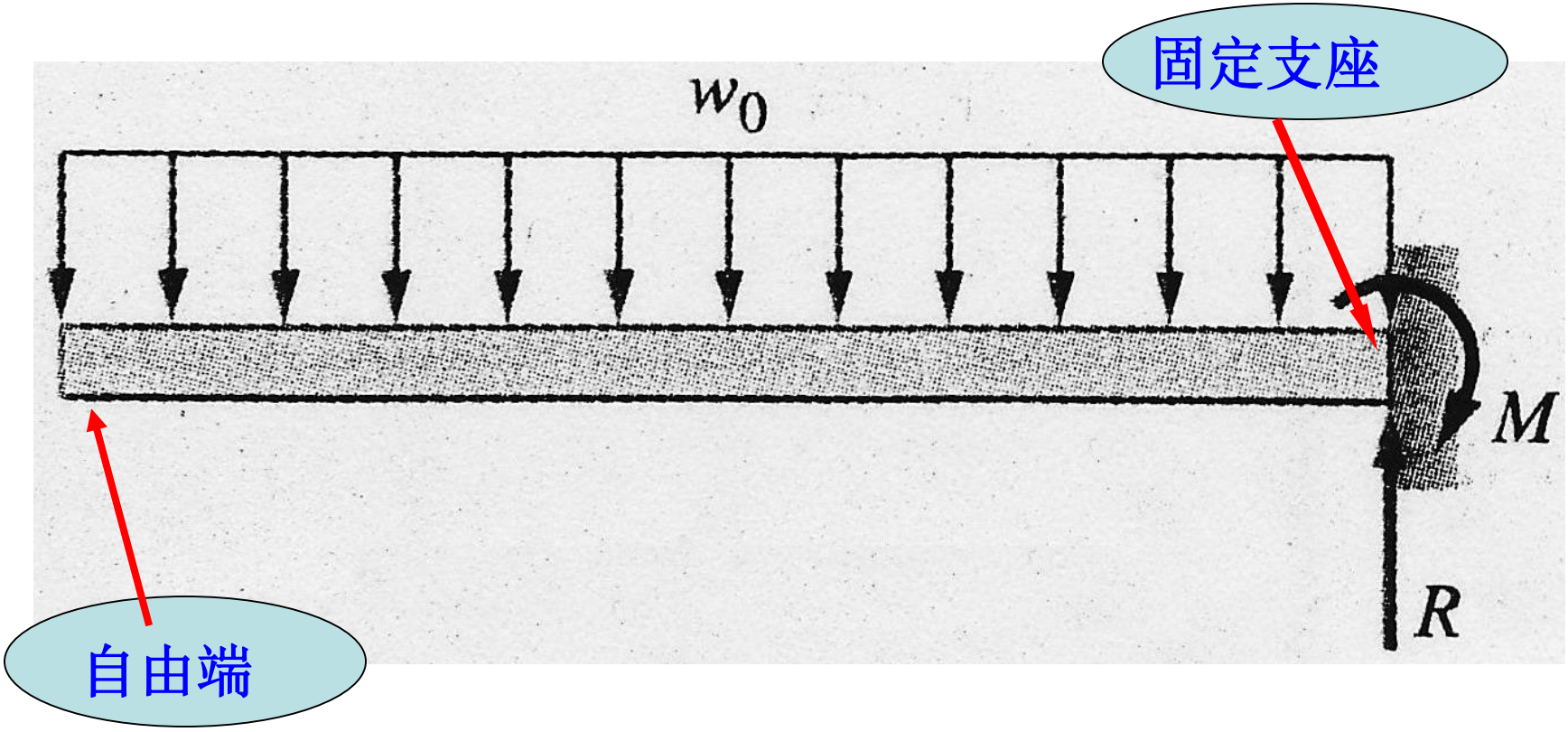
※ 静定梁的分类



简支梁

§ 2 静定梁的分类

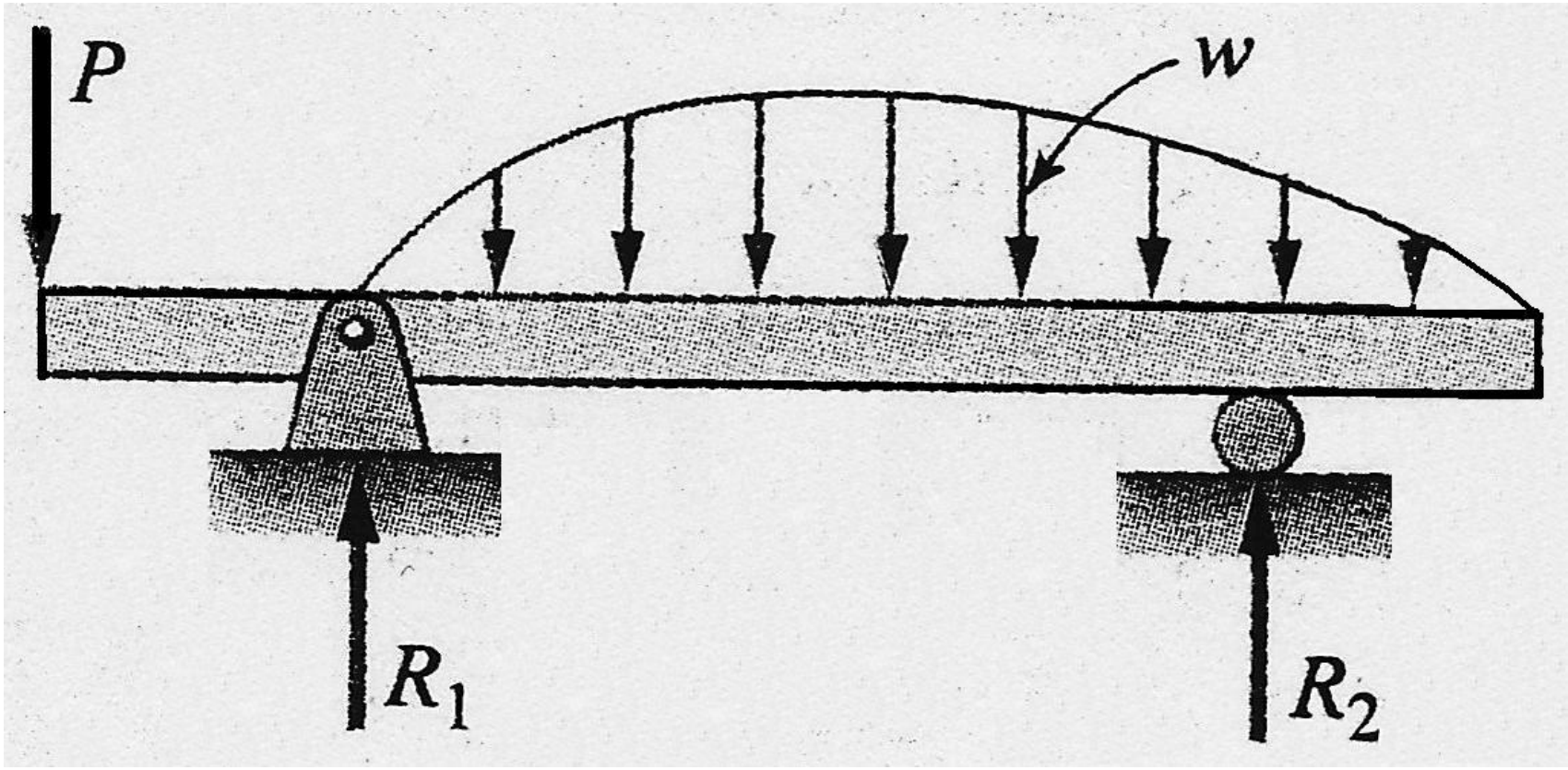
※ 静定梁的分类



悬臂梁

§ 2 静定梁的分类

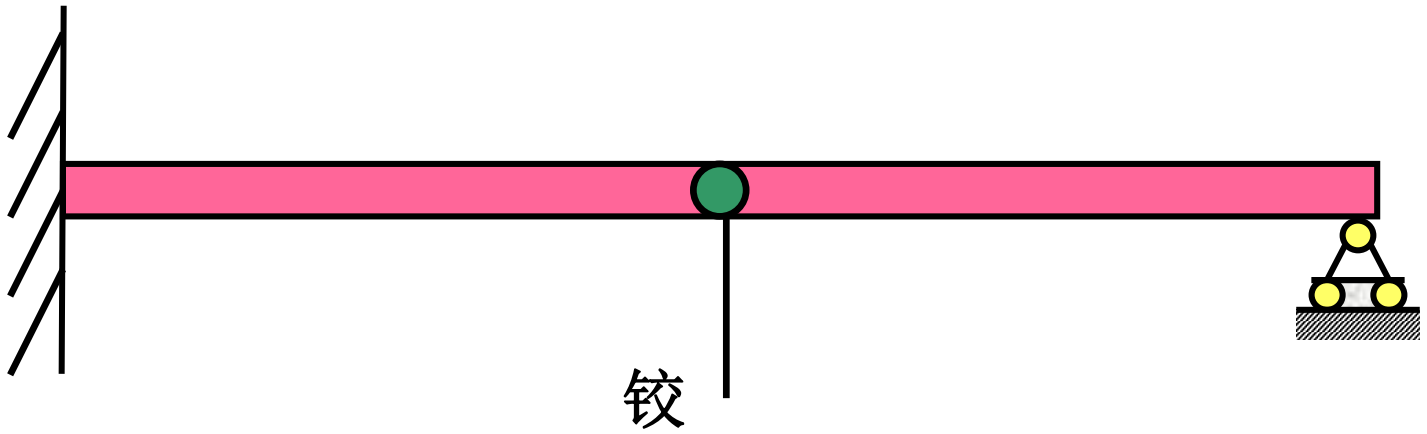
※ 静定梁的分类



外伸梁

§ 2 静定梁的分类

※ 静定梁的分类



连续梁

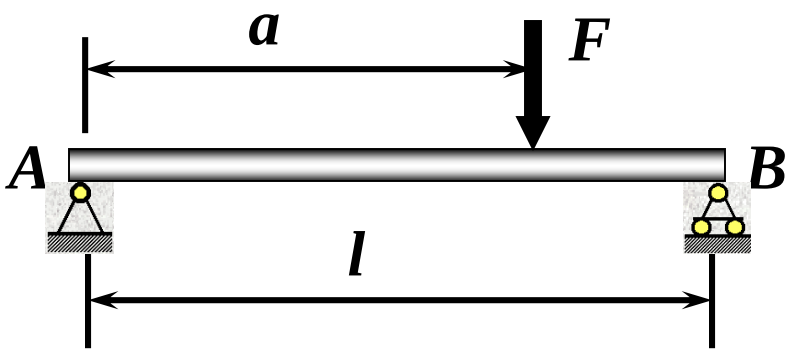
§ 3 剪力与弯矩 内力图

※ 梁的内力

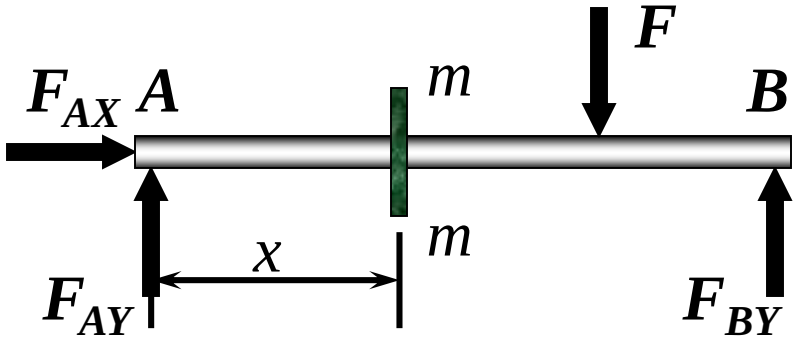
$$\sum F_X = 0,\ \ \therefore\ F_{AX} = 0$$

$$\sum \boldsymbol{M_A = 0},\ \ \boldsymbol{F_{BY} l - Fa = 0}$$

$$\sum F_Y = 0,\ \ F_{AY} - F + F_{BY} = 0$$



$$F_{BY} = \frac{Fa}{l},\quad F_{AY} = \frac{F(l-a)}{l}$$

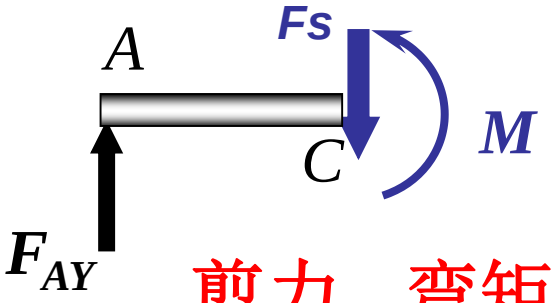


$$\sum F_Y = 0,\qquad F_{AY} - F_s = 0.$$

$$\boldsymbol{F_s = F_{AY} = \frac{F(l-a)}{l}}$$

$$\sum m_C = 0,\qquad M - F_{AY} x = 0.$$

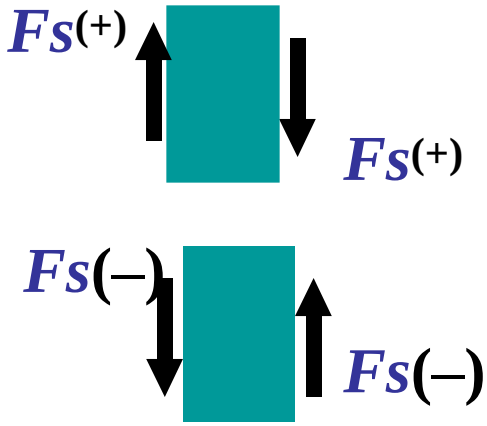
$$M = F_{AY} x = \frac{F(l-a)}{l} x$$



剪力　弯矩

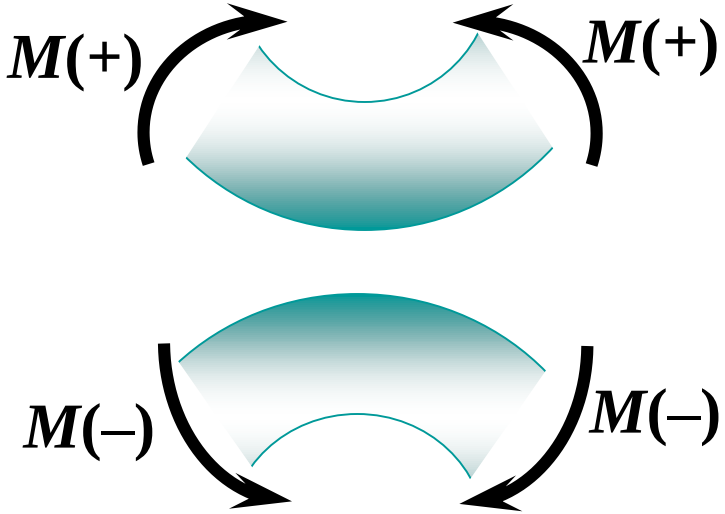
§ 3 剪力与弯矩 内力图

※ 剪力和弯矩的符号规定



- 剪力：令脱离体顺时针转动为正，
  反之为负

- 弯矩： 令梁变形为向上弯曲为 正 ，
  反之为负
  (笑脸为正，哭脸为负).

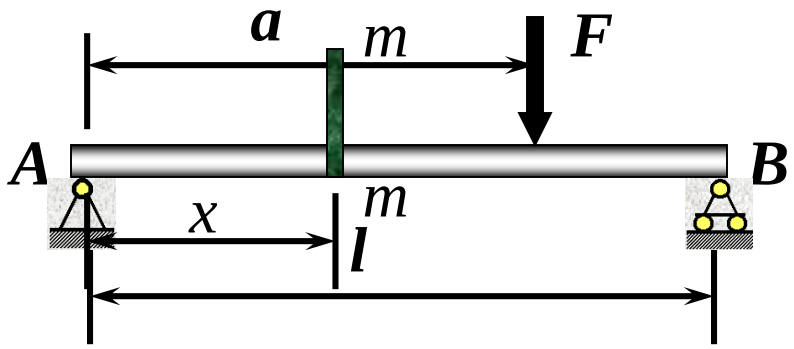


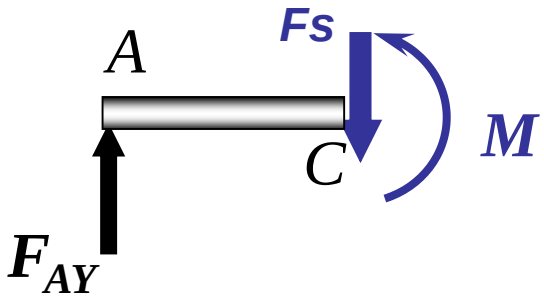
※ 剪力和弯矩的符号规定

取 *AC* 为脱离体

$$F_s = F_{AY} = \frac{F(l-a)}{l}$$

$$M = F_{AY}x = \frac{F(l-a)}{l}x$$



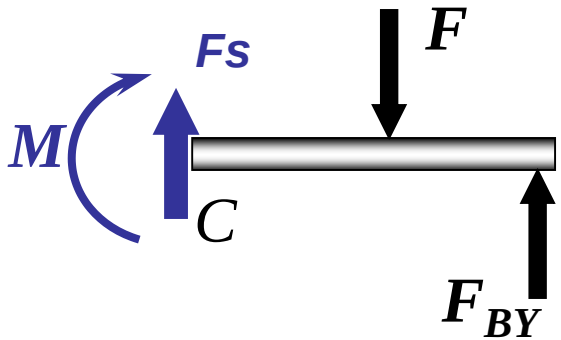


取*CB* 为脱离体

$$\sum Y = 0, \quad F_s - F + F_{BY} = 0.$$

$$\sum m_C = 0,$$

$$F_{BY}(l-x) - F(a-x) - M = 0.$$



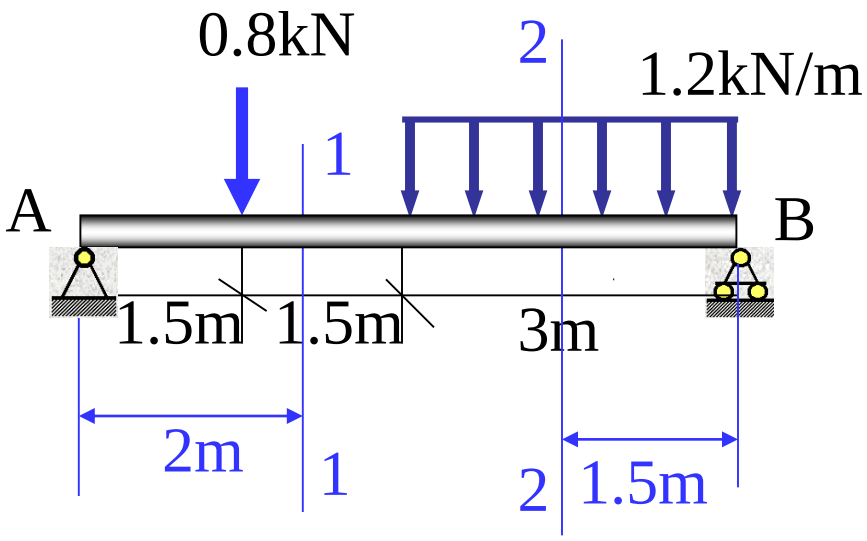
$$F_s = \frac{F(l-a)}{l}, \quad M = \frac{F(l-a)}{l}x$$

相同的大小和方向

§ 3 剪力与弯矩 内力图

例 1

确定截面 1-1 和 2-2 的内力



解：

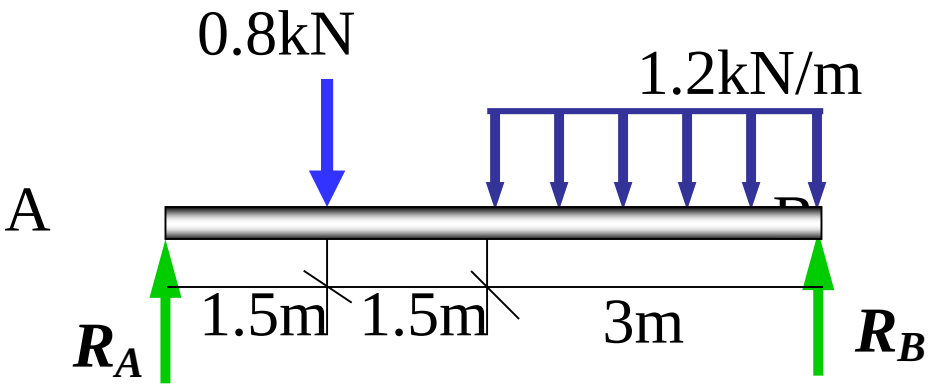
1、支反力

$$\sum F_Y = 0,$$

$$R_A + R_B - 0.8 - 1.2\times 3 = 0$$

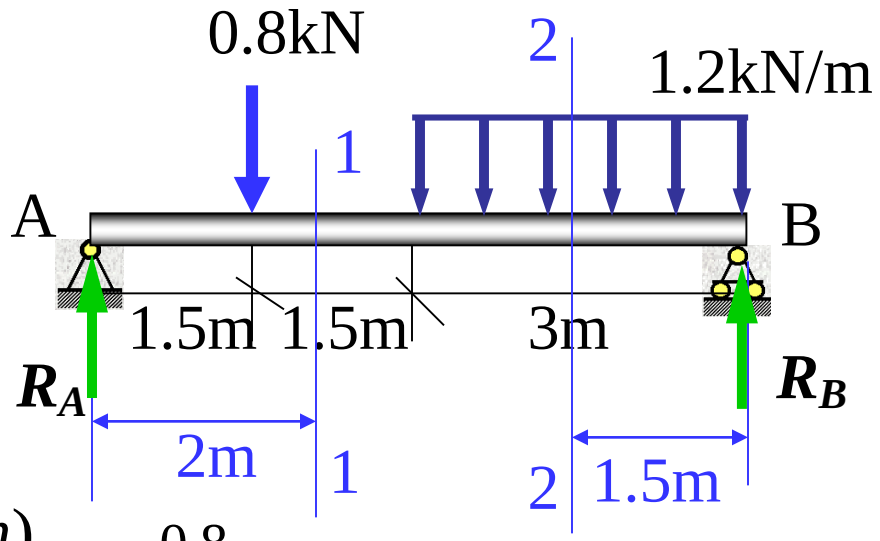
$$\sum M_B = 0,$$

$$1.2\times 3\times 1.5 + 0.8\times 4.5 - R_A\times 6 = 0$$



$$R_A = 1.5\,(kN), \quad R_B = 2.9\,(kN)$$

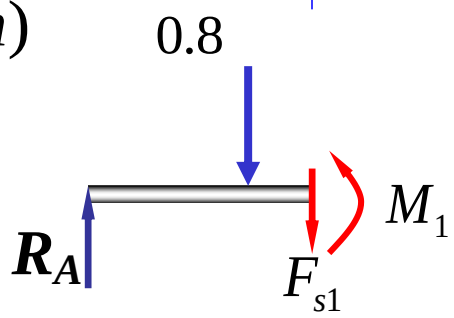
2、1-1截面的内力

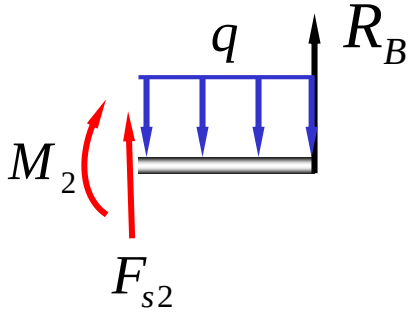


$$F_{s1} = R_A - 0.8$$
$$= 1.5 - 0.8 = 0.7\ (kN)$$

$$M_1 = R_A \times 2 - 0.8 \times 0.5$$
$$= 1.5 \times 2 - 0.8 \times 0.5 = 2.6\ (kN \cdot m)$$

3、2-2 截面的内力





$$F_{s2} = 1.2 \times 1.5 - 2.9 = -1.1(kN)$$

$$M_2 = R_B \times 1.5 - 1.2 \times 1.5 \times 0.75$$

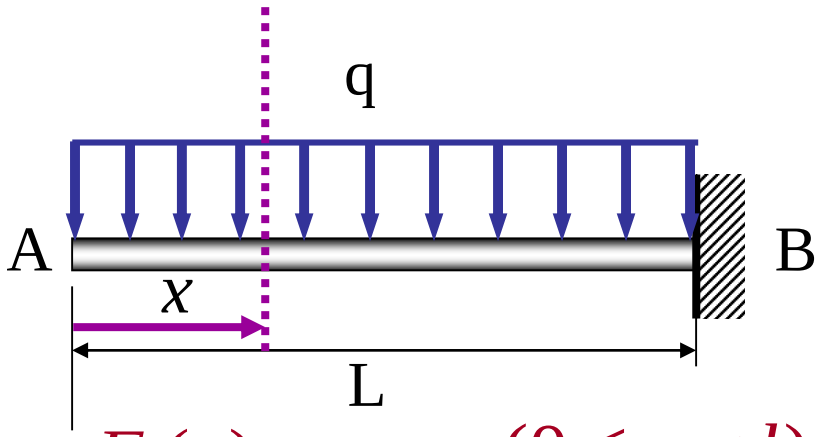
$$= 2.9 \times 1.5 - 1.2 \times 1.5 \times 0.75 \quad = 3.0(kN \cdot m)$$

§ 3 剪力与弯矩 内力图

※ 剪力图和弯矩图

剪力方程 $F_S = F_S(x)$

弯矩方程 $M = M(x)$

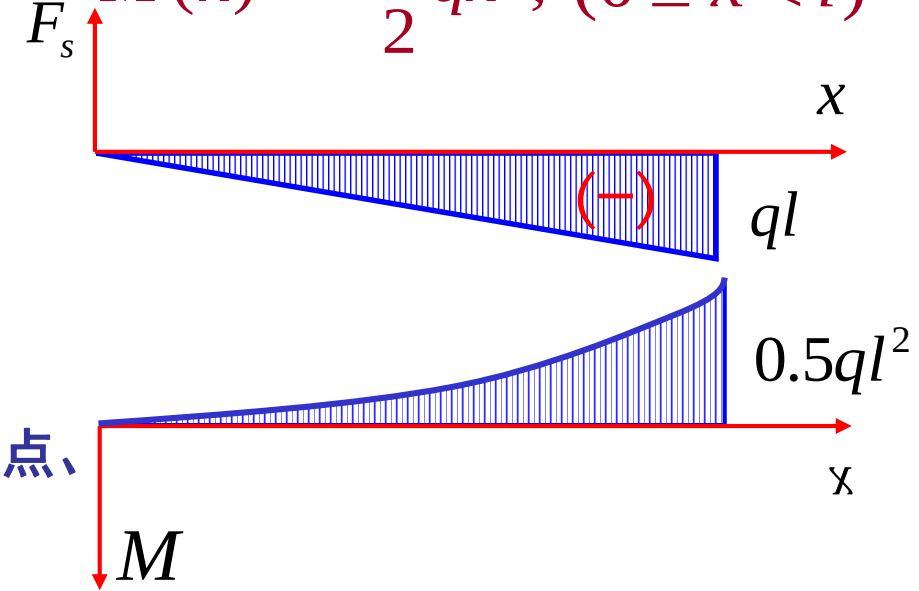


反映梁的横截面上的剪力和弯矩随截面位置变化的函数式。

$$F_s(x) = -qx, \quad (0 \le x < l)$$

$$M(x) = -\frac{1}{2}qx^2, \quad (0 \le x < l)$$

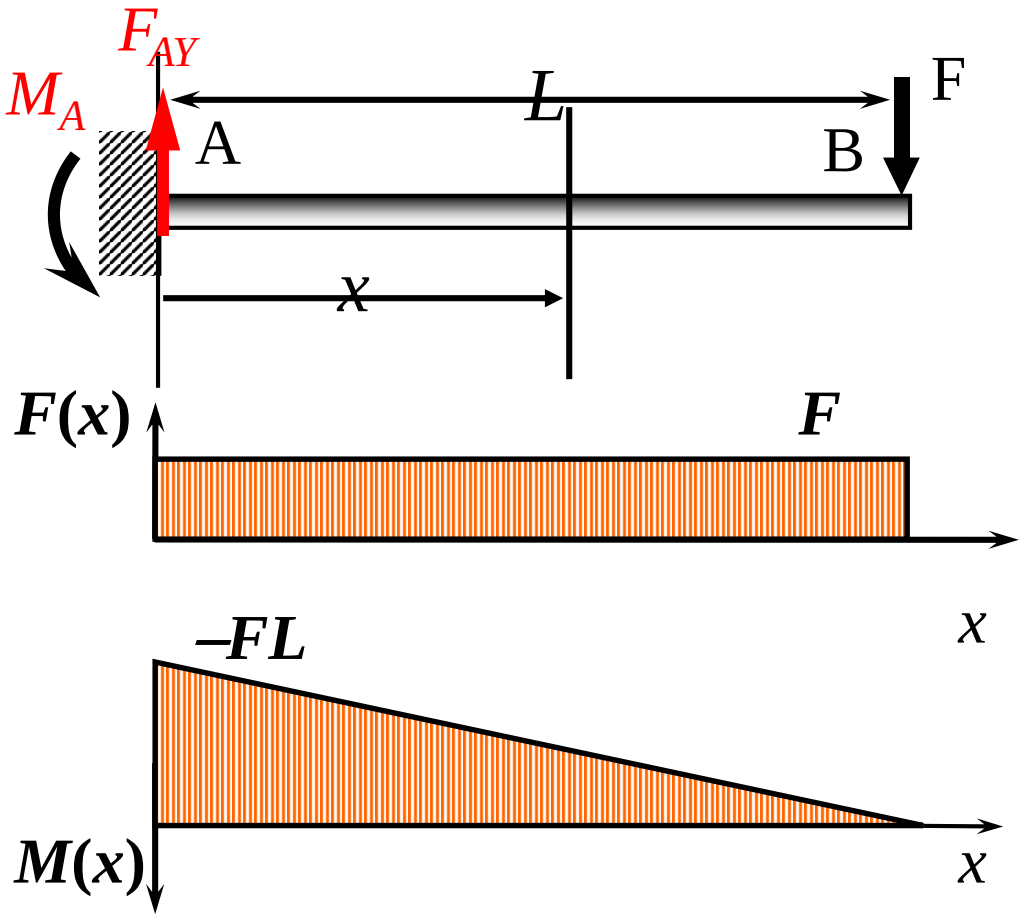
显示剪力和弯矩随截面位置的变化规律的图形则分别称为**剪力图**和**弯矩图**。



注意：不能用一个函数表达的要分段，分段点为：集中力作用点、集中力偶作用点、分布力的起点、终点。

例 2

画出图中梁的内力图



解：

1、求支反力

$$F_{AY}=F\ ;\ M_A=FL$$

2、内力方程

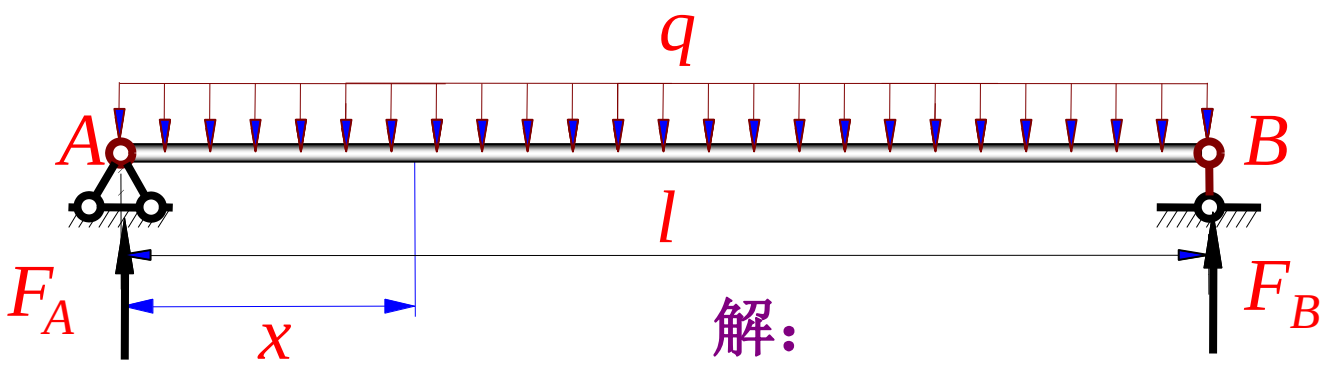
$$F_s(x)=F_{AY}=F \qquad (0<x<l)$$

$$M(x)=F_{AY}x-M_A$$
$$=F(x-L) \qquad (0<x\le l)$$

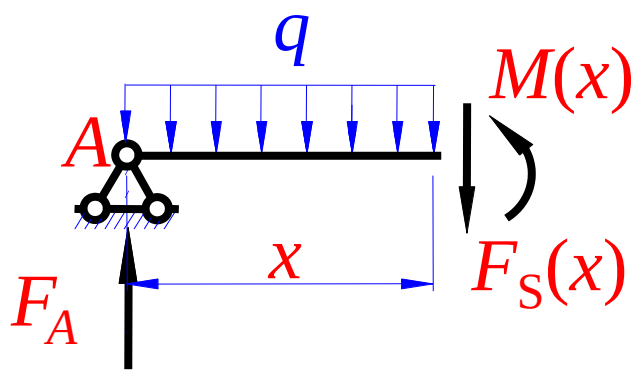
3、画内力图

例 3

画出图中梁的内力图



解：



1、求支反力

$$F_A = F_B = \frac{ql}{2}$$

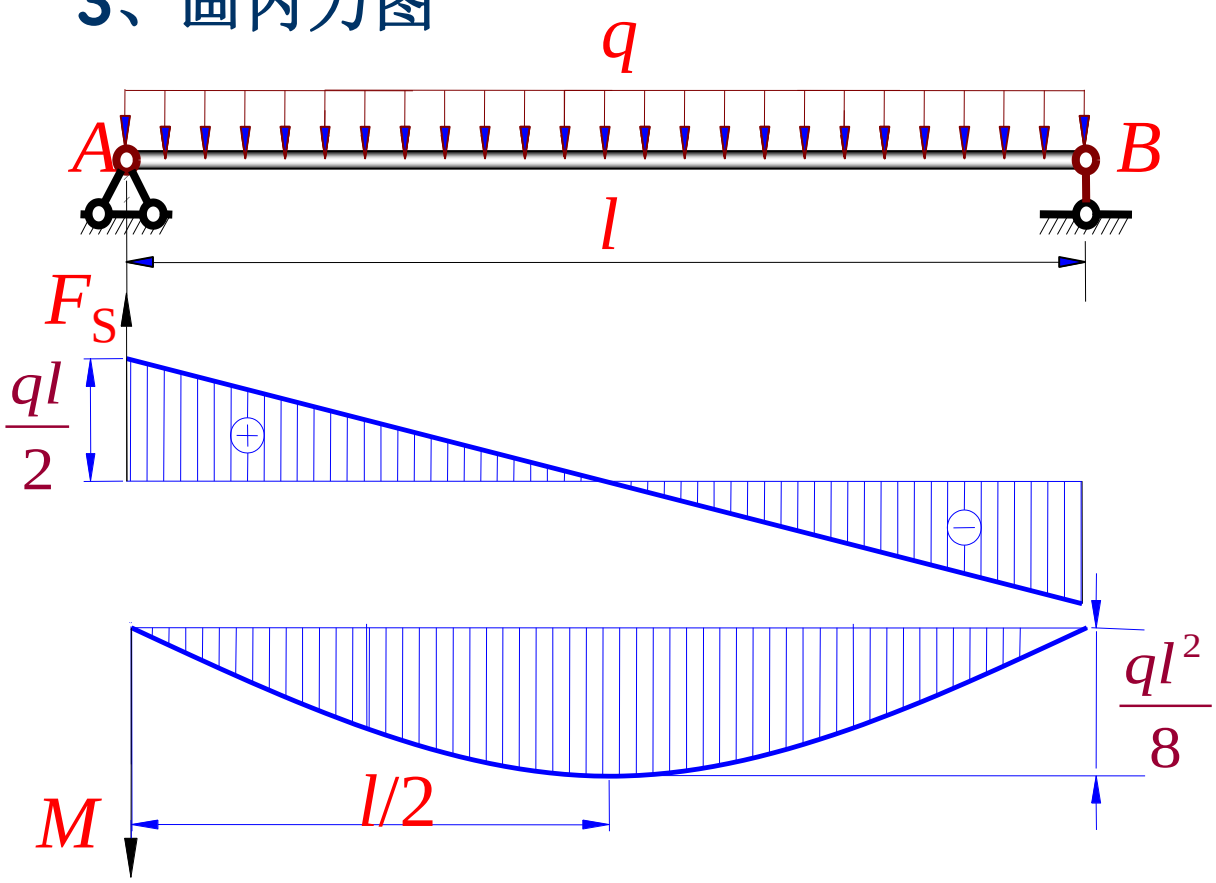
2、内力方程

$$F_S(x) = F_A - qx = \frac{ql}{2} - qx$$

$$M(x) = F_A x - qx \times \frac{x}{2} = \frac{qlx}{2} - \frac{qx^2}{2}$$

§ 3 剪力与弯矩 内力图

3、画内力图



$$F_S(x)=\frac{ql}{2}-qx$$

$$M(x)=\frac{qlx}{2}-\frac{qx^2}{2}$$

$$\left|F_{S,max}\right|=\frac{ql}{2}$$

$$M_{max}=\frac{ql^2}{8}$$

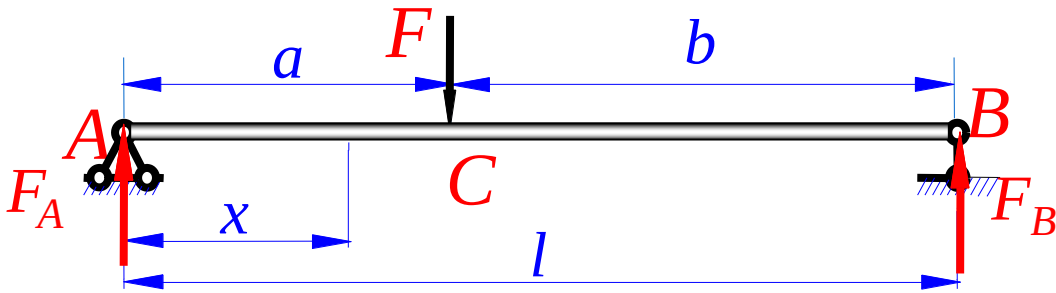
***结构对称、载荷对称, 则剪力图反对称，弯矩图对称**

***剪力为零的截面弯矩有极值。**

§ 3 剪力与弯矩 内力图

例 4 画出图中所示梁的内力图

解：1、支反力

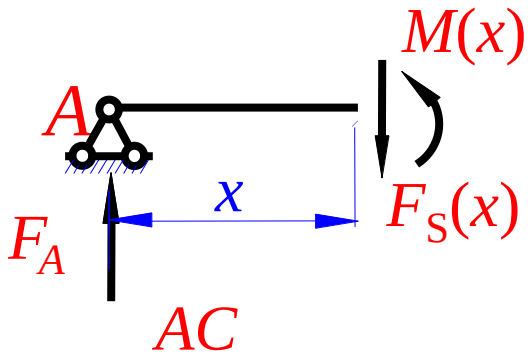


$$F_A=\frac{Fb}{l} \qquad F_B=\frac{Fa}{l}$$

2、内力方程

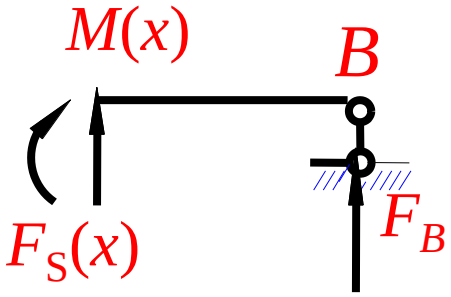
$$F_S(x)=\frac{Fb}{l}(0<x<a)$$

$$M(x)=\frac{Fb}{l}x(0\le x\le a)$$



$$F_S(x)=-F_B=-\frac{Fa}{l}(a<x<l)$$

$$M(x)=F_B(l-x)=\frac{Fa}{l}(l-x) \qquad (a\le x\le l)$$



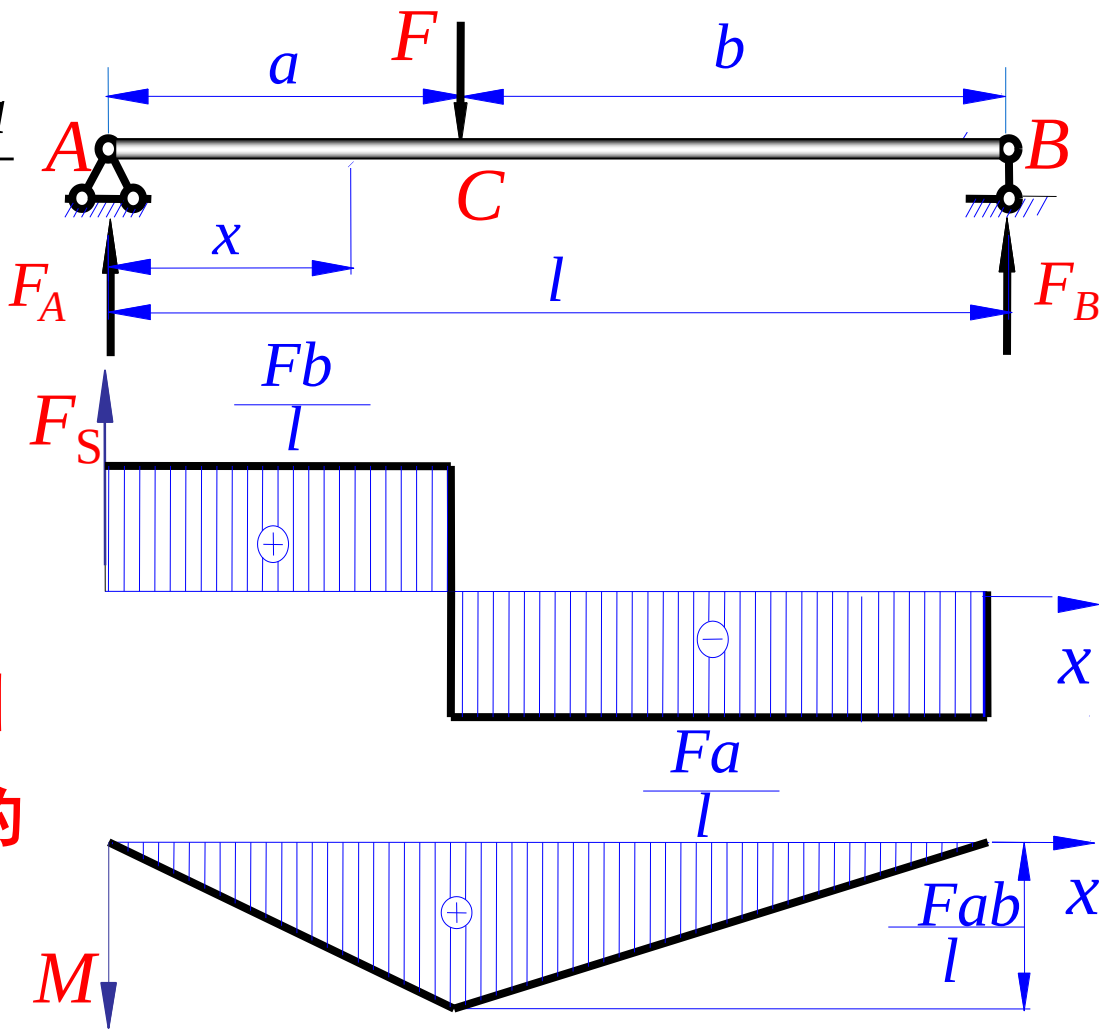
§ 3 剪力与弯矩 内力图

3、内力图

$$F_{S1}(x)=\frac{Fb}{l} \qquad F_{S2}(x)=-\frac{Fa}{l}$$

$$M_1(x)=\frac{Fb}{l}x$$

$$M_2(x)=\frac{Fa}{l}(l-x)$$



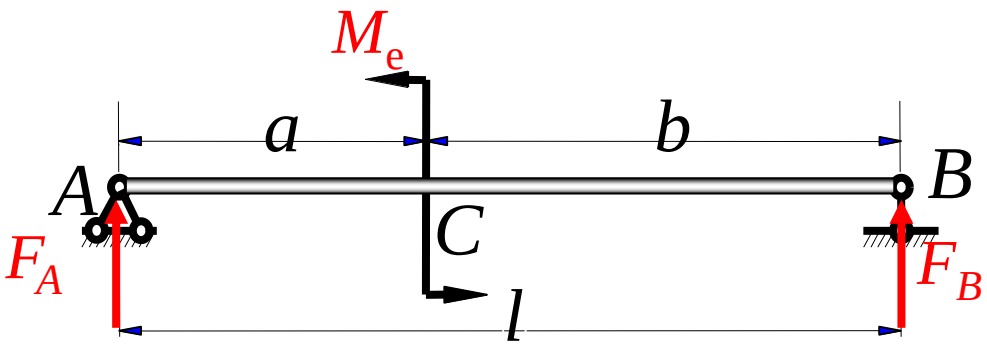
***在 集中力F 作用处，剪力图有突变，突变值为集中力的大小；弯矩图有转折。**

* $M_{\max}=\frac{Fl}{4}$，at $a=b=l/2$

例 5

画出图中所示梁的内力图

解：

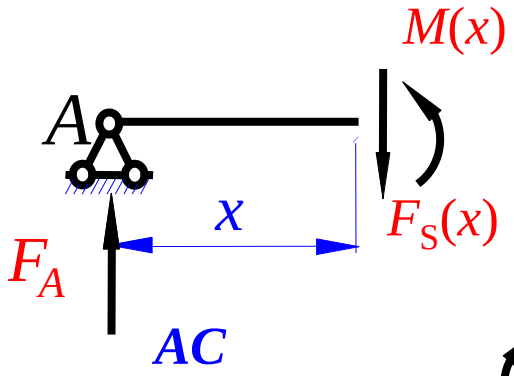


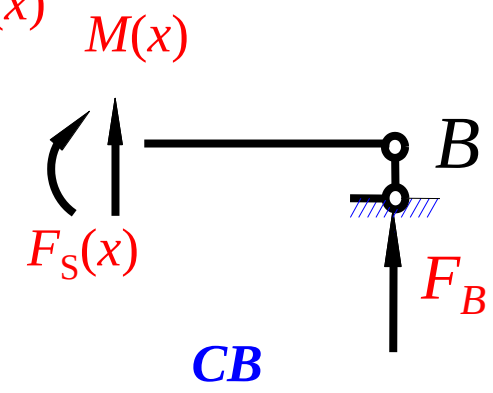
1、支反力

$$\sum M_B = 0 \quad M_e - F_A \times l = 0$$

$$F_A = \frac{M_e}{l}(\uparrow) \quad F_B = \frac{M_e}{l}(\downarrow)$$

2、内力方程



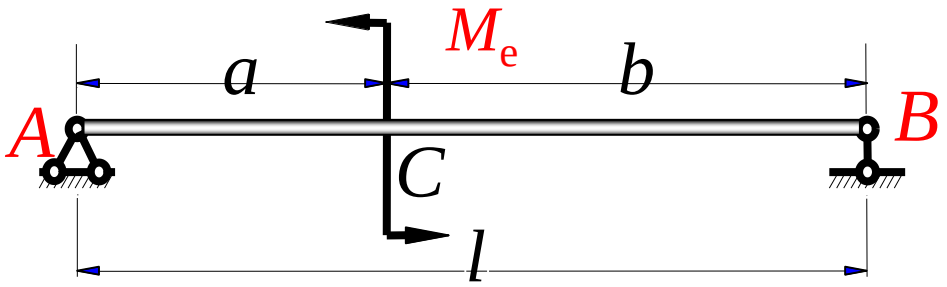


$$F_S(x) = F_A = \frac{M_e}{l}(0 < x < l)$$

$$M(x) = F_A x = \frac{M_e}{l} x \ (0 \le x < a)$$

$$M(x) = F_A x - M_e = -\frac{M_e}{l}(l - x) \ (a < x \le l)$$

§ 3 剪力与弯矩 内力图

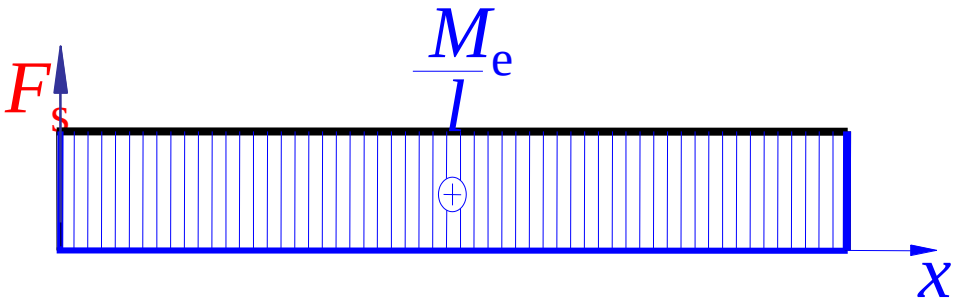


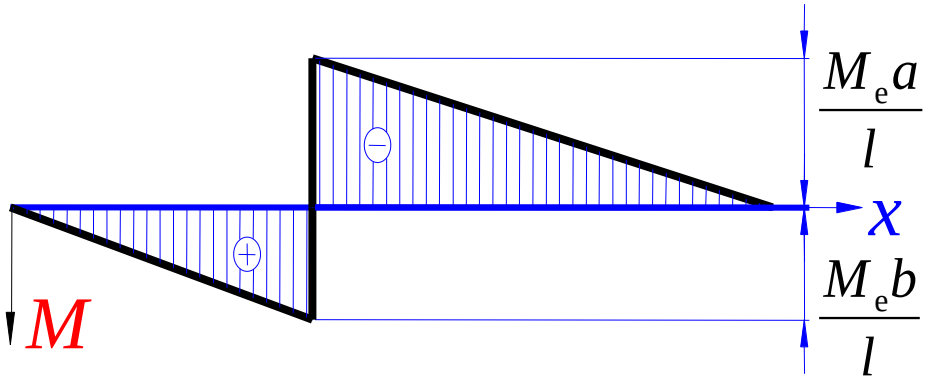
3、内力图

$$F_S(x)=\frac{M_e}{l}$$

$$M(x)=\frac{M_e}{l}x \qquad (0\le x<a)$$

$$M(x)=-\frac{M_e}{l}(l-x) \qquad (a<x\le l)$$





$b>a$时 $|M_{max}|=\frac{M_e b}{l}$

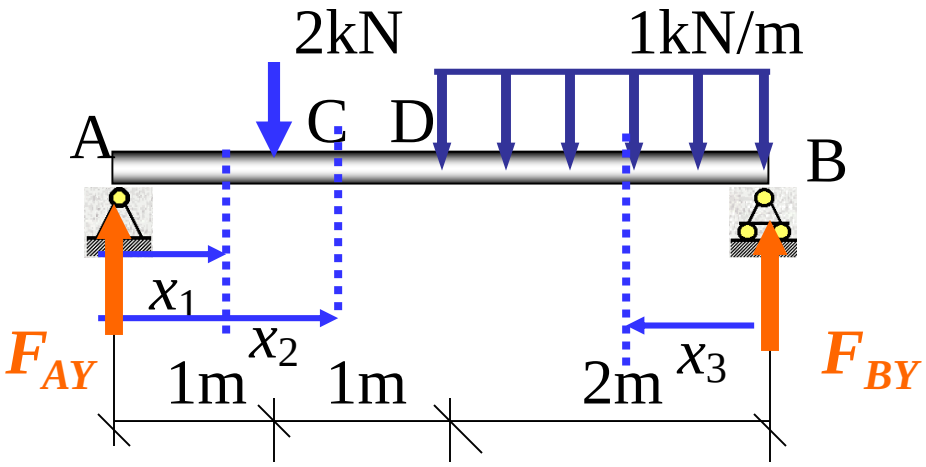
发生在C截面右侧。

集中力偶作用点处剪力图无影响，弯矩图有突变，突变值的大小等于集中力偶的大小。

§ 3 剪力与弯矩 内力图

例 6

画出图中梁的内力图



解：

1、支反力

$$\sum Y = 0, \quad F_{AY} + F_{BY} - 2 - 1\times 2 = 0$$

$$\sum M_B = 0, \; 1\times 2\times 1 + 2\times 3 - F_{AY}\,4 = 0$$

$$\therefore \quad F_{AY} = 2(kN); \qquad F_{BY} = 2(kN)$$

2. 内力方程

AC：

$$F_s(x_1) = F_{AY} = 2(kN)$$

$$M(x_1) = F_{AY}x_1 = 2x_1(kN.m)$$

$$F_s(x_2) = F_{AY} - 2 = 2 - 2 = 0,$$

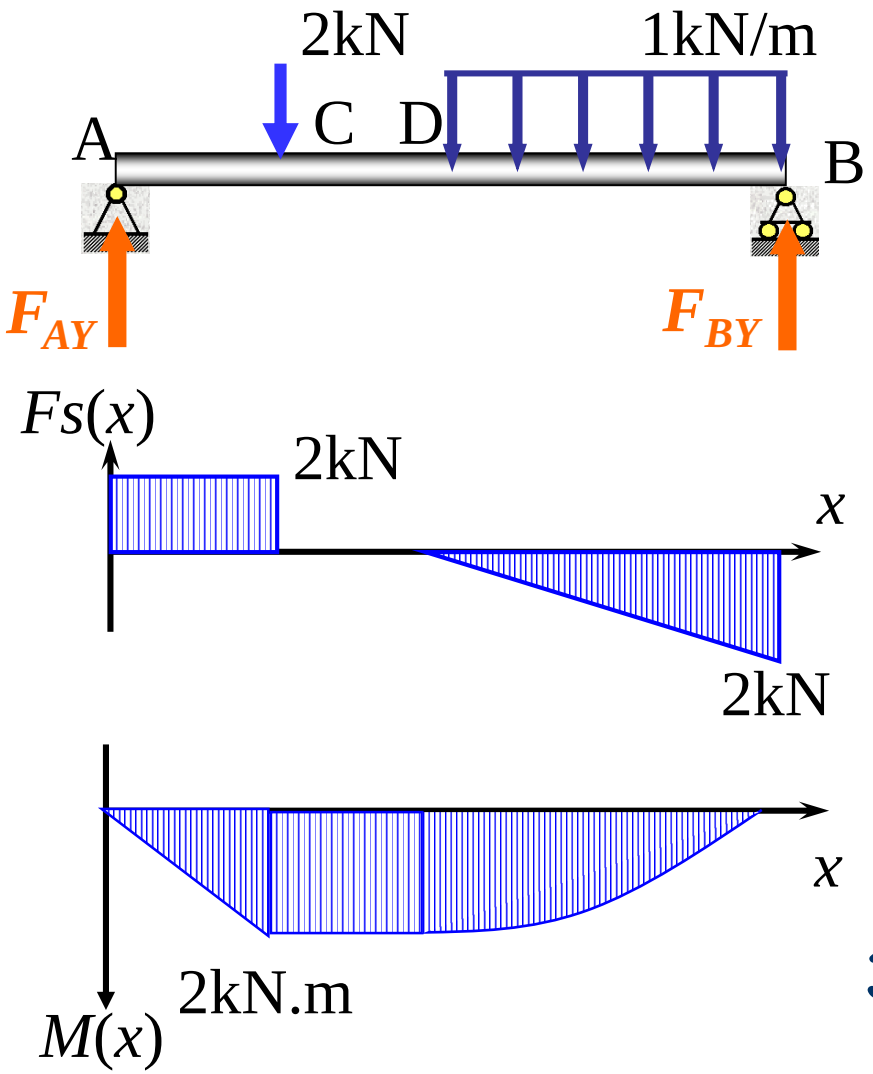
$$M(x_2) = F_{AY}x_2 - 2(x_2 - 1) = 2(kN.m)$$

***BD*：**

$$F_s(x_3) = -F_{BY} + 1\times x_3 = -2 + x_3$$

$$M(x_3) = F_{BY}x_3 - 1\times x_3 \times \frac{x_3}{2} = 2x_3 - \frac{x_3^2}{2}$$

§ 3 剪力与弯矩 内力图



$$AC: F_s(x_1)=2, \quad (0<x_1<1)$$

$$M(x_1)=2x_1, \quad (0\le x_1\le 1)$$

$$CD: F_s(x_2)=0, \quad (1<x_2<2)$$

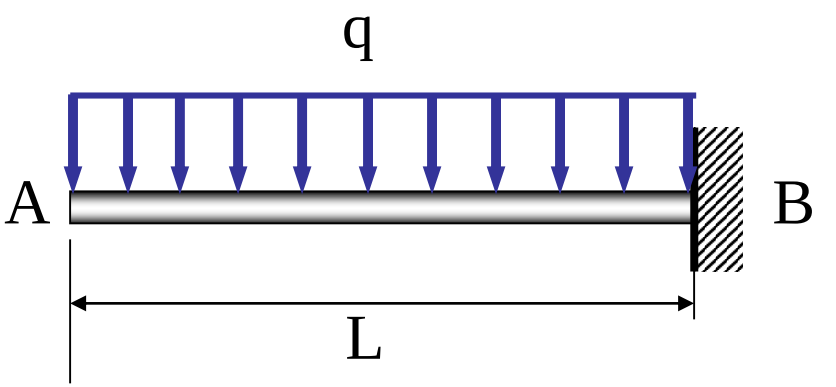
$$M(x_2)=2, \quad (1\le x_2\le 2)$$

$$BD: F_s(x_3)=-2+x_3, \quad (0<x_3\le 2)$$

$$M(x_3)=2x_3-\frac{x_3^2}{2}, \quad (0\le x_3\le 2)$$

3、内力图

§4 剪力、弯矩和分布荷载集度间的关系

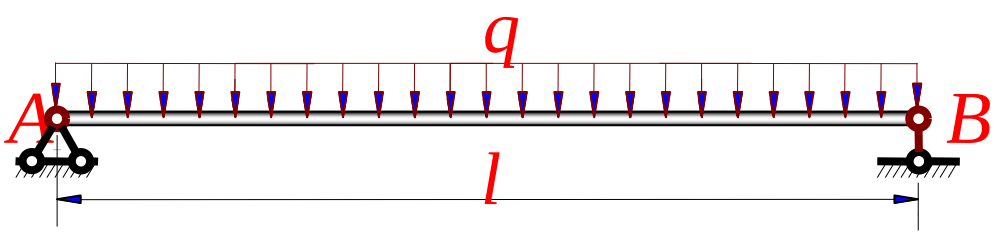


$$F_s(x) = -qx, \quad (0 \le x < l)$$

$$M(x) = -\frac{1}{2}qx^2, \ (0 \le x < l)$$

$$\frac{dF_s(x)}{dx} = -q \quad = q(x)$$

$$\frac{dM(x)}{dx} = -qx \quad = F_s(x),$$



$$F_S(x) = F_A - qx = \frac{ql}{2} - qx$$

$$M(x) = F_A x - qx \times \frac{x}{2} = \frac{qlx}{2} - \frac{qx^2}{2}$$

$$\frac{dF_s(x)}{dx} = -q \quad = q(x)$$

$$\frac{dM(x)}{dx} = \frac{1}{2}ql - qx \quad = F_s(x),$$